ADVANCED DYNAMICS — PHY 4241/5227

HOME AND CLASS WORK – SET 7

(February 12, 2010)

(23) Assume a central potential and $\vec{L} \neq 0$.

1. Use initial conditions in the CM system, $\vec{r}_0$ and $\vec{v}_0$ to define unit vectors $\hat{e}_1$, $\hat{e}_2$ and $\hat{e}_3$ of a Cartesian coordinates system in which

   $$L = |\vec{L}| = \mu\, r^2\, \dot{\phi}\,, \quad \phi_0 = 0\,,\; \dot{\phi}_0 > 0$$

   holds (ϕ azimuth angle). Due February 12 in class (4 points).

2. Express the total energy $E_{\rm cm}$ in terms of r and ϕ and their derivatives (i.e., get rid of $\vec{v}$). Substitute the conserved angular momentum L into your expression. Combine all terms, which contain r but not $\dot{r}$, into the definition of an effective potential $U_{\rm eff}(r)$. Due February 12 in class (4 points).

(24) Use (in arbitrary units) the parameters $\alpha = \mu = L = 1$ to plot the potential and the effective potential of the Kepler problem

$$U = -\frac{\alpha}{r} \;\; \text{and} \;\; U_{\rm eff}(r) = U(r) + \frac{L^2}{2\,\mu\, r^2}$$

in the range $0 < r \leq 10$ with the ordinate restricted to the interval $[-1, 0.6]$. Describe the motion for $E_{\rm cm} = U^{\rm min}_{\rm eff}$, where $U^{\rm min}_{\rm eff} = U_{\rm eff}(r^{\rm min})$ is the minimum of the effective potential. Calculate $r^{\rm min}$ as function of L, μ and α and give then the numerical value. Due February 15 before class (8 points).

(25a) Separate variables in the equation

$$E_{\rm cm} = \frac{\mu}{2}\dot{r}^2 + U_{\rm eff}(r)$$

to derive an integral equation for $t(r)$. Due February 15 in class (4 points).

(25b) Use angular momentum conservation and the separation of variables in the previous problem to derive an integral equation for $\phi(r)$. Due February 15 in class (4 points).