ADVANCED DYNAMICS — PHY 4241/5227

HOME AND CLASS WORK – SET 8

(February 19, 2010)

(27a) Kepler problem: Use unit with $G = 1$ for the gravitational constant and the initial conditions given in the table below (you can download the data in the file `Korbits.txt`) to plot the four orbits in the coordinate system used in Eq. (39) of the Notes. Make first a table of L, E, p, e. Then, put all orbits in one plot. Hint: For the first case $e = 0.138826$. Due February 24 before class (10 points).

(27b) Get the initial positions into the plot of the previous homework. February 26 before class (4 extra points).

(27c) Find positions and velocities in the original system at time $t = 1.5$ (arbitrary units and $G = 1$). Ten extra points when turned in by March 15 or earlier.

	Masses		Initial Positions			Initial Velocities		
#	i	m_i	$x^1_{i,0}$	$x^2_{i,0}$	$x^3_{i,0}$	$\dot{x}^1_{i,0}$	$\dot{x}^2_{i,0}$	$\dot{x}^3_{i,0}$
1	1	0.651	0.585	-0.238	-0.755	-0.828	-0.865	-0.726
	2	0.931	-0.096	0.000	0.357	-0.209	0.107	-0.660
2	1	1.510	0.460	-0.359	-0.234	-0.918	-0.941	-0.323
	2	0.126	-0.066	-0.090	-0.809	0.789	0.788	0.620
3	1	1.328	-0.125	0.898	0.194	-0.452	0.172	0.125
	2	1.999	-0.449	-0.085	-0.454	-0.976	-0.990	-0.968
4	1	0.180	0.204	-0.968	-0.753	-0.811	-0.632	0.784
	2	1.560	-0.889	-0.979	0.854	-0.323	-0.774	-0.533

Table 1: Initial conditions for the Kepler problem (arbitrary units and $G = 1$).

(28) Assume $0 < e < 1$ for the eccentricity and transform the elliptic equation

$$\frac{p}{r} = 1 + e\,\cos(\phi)\,, \quad p > 0 \;\text{ into the form }\; \frac{x'^2}{a^2} + \frac{y'2}{b^2} = 1\ .$$

This means, *derive* the definitions of x', y', major half-axis a and minor half-axis b in terms of x, y, p and e (p and e are Landau-Lifshitz notation in M&T it is α and ϵ). Due February 19 in class 4 points.

(29) Discuss the $\vec{L} = 0$ case of the Kepler problem using $r = |\vec{r}|$ as defined before. Due February 19 in class 4 points.

1. Are there turning points? Sketch the potential together with assumed $E_{\rm cm}$ energy values.
2. For which initial conditions and energies $E_{\rm cm}$ will the objects collide or not collide?
3. Find $r(t)$ for the $E_{\rm cm} = 0$ case.