**ADVANCED DYNAMICS — PHY 4241/5227**

**HOME AND CLASS WORK – SET 12**

(March 12, 2010)

(42) Derive the Taylor expansions of the functions $\exp(x)$, $\cosh(x)$, $\cos(x)$, $\sinh(x)$ and $\sin(x)$ about $x = 0$ from the equation

$$f(x) = \sum_{n=0}^{\infty} f^{(n)}(0)\,\frac{x^n}{n!}$$

by calculating $f^{(n)}(0)$ for each case explicitly. Do NOT use Euler's formula. Due April 2 before class (5 points).

(43) The matrix $L$ is defined by

$$L \;=\; \begin{pmatrix} l^0{}_0 & l^0{}_1 & l^0{}_2 & l^0{}_3 \\ l^1{}_0 & l^1{}_1 & l^1{}_2 & l^1{}_3 \\ l^2{}_0 & l^2{}_1 & l^2{}_2 & l^2{}_3 \\ l^3{}_0 & l^3{}_1 & l^3{}_2 & l^3{}_3 \end{pmatrix}$$

1. Calculate $-g\,L$ (2 points).
2. Write down the transpose matrix $\tilde{L}$ (2 points).
3. Calculate $\tilde{L}\,g$ (2 points).
4. Compare 1. and 2. to find the general form of $L$ (*i.e.* use $\tilde{L}\,g = -g\,L$, 4 points).
5. Obtain the same result by discussing the elements of the equation $g^{\alpha\beta}\tilde{l}_\beta{}^\gamma g_{\gamma\delta} = -l^\alpha{}_\delta$ (4 extra points. Hint: Do the contractions first.)

Due April 7 in class.

(44) In the following $K_1$ and $S_3$ are generators as defined in class.

1. Calculate $\exp(-\zeta\,K_1)$ and explain its physical meaning (3 points).
2. Calculate $\exp(-\phi\,S_3)$ and explain its physical meaning (3 points).

Due April 12 before class.

(45) Consider the spaceship journey again and plot versus the proper time $\tau$ the following quantities (due April 5 before class, 10 points):

1. The time on earth at which the spaceship, seen from earth, is at its position at time $\tau$.
2. The distance from earth as seen from earth.
3. The date of news received from earth, which is transmitted at the speed of light.