ADVANCED DYNAMICS — PHY 4241/5227 HOME AND CLASS WORK – SET 14

(April 8, 2010)

- (51) An electron and a positron, each with mass equal to 0.511 MeV, annihilate at rest into two photons. Chose the rest frame of the positronium, such that one of the photons moves in positive $x = x^1$ direction.
 - 1. For each photon find the momentum 4-vector.
 - 2. Find the 4-momentum in a frame that moves at a velocity $c\beta \hat{x}$ with respect to the rest frame of the positronium.
 - 3. Suppose that the annihilation took place 10^9 years ago in a galaxy that is receding from us at $\beta = 4/5$. What is the energy of the photon that we observe?

Due April 21 before class (10 points).

(52) Let $T^{\alpha\beta}$ be a rank two tensor.

- 1. Write down $T_S^{\alpha\beta}$, the symmetric part of $T^{\alpha\beta}$. Due April 19 in class (1 point).
- 2. Write down $T_A^{\alpha\beta}$, the antisymmetric part of $T^{\alpha\beta}$. Due April 19 in class (1 point).
- 3. Calculate $T_A^{\alpha\beta} T_{S\alpha\beta}$. Due April 19 in class (1 point).
- 4. Derive the continuity equation from the antisymmetry of $F^{\alpha\beta}$. Due April 19 in class (1 point).
- 5. Show $\partial_{\alpha} * F^{\alpha\beta} = 0$, where

$${}^{*}F^{\alpha\beta} = \frac{1}{2}\epsilon^{\alpha\beta\gamma\delta}F_{\gamma\delta}$$

is the dual tensor. Here $\epsilon^{\alpha\beta\gamma\delta}$ is the completely antisymmetric Levi-Cevita tensors: +1 for $(\alpha, \beta, \gamma, \delta)$ and even permutation of (0, 1, 2, 3), -1 for $(\alpha, \beta, \gamma, \delta)$ and even permutation of (0, 1, 2, 3). Due April 19 in class (2 points).

(53) Under a gauge transformation the vector potential transform according to

$$A^{\alpha} \to A'^{\alpha} = A^{\alpha} + \partial^{\alpha} \Lambda$$

Calculate the corresponding transformation of the electromagnetic field tensor

$$F^{\alpha\beta} \to F'^{\,\alpha\beta}$$
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Due April 21 before class (4 points).