

MIDTERM ADVANCED DYNAMICS — PHY-4241/5227

March 6, 2009

PROBLEM 1 (20 points)

The Lagrangian of the 1D harmonic oscillator is

$$L = \frac{1}{2} m \dot{x}^2 - \frac{1}{2} k x^2 .$$

1. Find the momentum p .
2. Write down the Hamiltonian $H(p, x)$.
3. Write down Hamilton's equations.
4. Show that Hamilton's equations give Newton's force law.

PROBLEM 2 (20 points)

1. Use spherical coordinates (θ, ϕ, r) to write down the Lagrangian for a particle of mass m moving in the presence of a spherically symmetric potential $V(\mathbf{r}) = V(r)$.
2. Write down the Euler-Lagrange equations for the two angles.
3. Use the Lagrangian to identify two conserved quantities.
4. Find $\phi(t)$ for the solution with the initial condition $\dot{\phi}(0) = 0$ where ϕ is the azimuth angle.

PROBLEM 3 (5 points)

Write down the four Maxwell's equations (in vacuum) in the presence of a charge density ρ and a current density \mathbf{J} (you may use SI or Gaussian units).

PROBLEM 4 (15 points)

1. Write down the two basic postulates of the Special Theory of Relativity.
2. The lifetime of a pion is $\tau = 2.6 \times 10^{-8}$ s (with respect to its rest frame). Assume the pion travels in a collider experiment at 99% of the speed of light with respect to the Lab frame.
 - (a) How long is the observed lifetime of the pion in the Lab frame?
 - (b) How far does the pion travel in the Lab frame during one lifetime?