## ADVANCED DYNAMICS — PHY 4241/5227 HOME AND CLASS WORK – SET 2

Solution for assignment 10.

## 1. The Lagrangian is

$$L = \frac{1}{2} \dot{x}^2 - \frac{1}{2} x^2 \; .$$

Thus, the Euler-Lagrange equation becomes

$$0 = \frac{\partial L}{\partial x} - \frac{d}{dt} \frac{\partial L}{\partial \dot{x}} = -x - \ddot{x}$$

and the general solution is given by  $x(t) = A \sin(t) + B \cos(t)$ . The integration constants are determined by the initial conditions

$$x(0) = B = 0$$
 and  $x(\pi/2) = A = 1$ 

so that the exact path is

$$x(t) = \sin(t) \; .$$

For this path the action is given by

$$S[x(t)] = \frac{1}{2} \int_0^{\pi/2} dt \left[ \cos^2(t) - \sin^2(t) \right] = \frac{1}{2} \int_0^{\pi/2} dt \, \cos(2t) = 0 \; .$$

2. For a linear path x(t) = a + bt subject to the boundary conditions:

$$x(0) = a = 0$$
 and  $x(\pi/2) = b \frac{\pi}{2} = 1 \implies b = \frac{2}{\pi}$ 

so that this path is

$$x(t) = \frac{2}{\pi}t \; .$$

For this path the action is given by

$$S[x(t)] = \frac{1}{2} \int_0^{\pi/2} dt \left[ \left(\frac{2}{\pi}\right)^2 - \left(\frac{2}{\pi}\right)^2 t^2 \right] = \frac{2}{\pi^2} \int_0^{\pi/2} dt \left(1 - t^2\right)$$
$$= \frac{2}{\pi^2} \left[ \frac{\pi}{2} - \frac{1}{3} \left(\frac{2}{\pi}\right)^3 \right] = \frac{1}{\pi} \left(1 - \frac{\pi^2}{12}\right) = \frac{1}{\pi} - \frac{\pi}{12} = 0.0565 \dots$$

3. Assuming that the previous result is in  $J \cdot s$ , we find for the large number

$$S/\hbar\approx 5.37\times 10^{32}$$
 .