ADVANCED DYNAMICS — PHY 4241/5227

HOME AND CLASS WORK – SET 3

Solution for assignment 15a: Motivation of generalized momentum definition.

$$\frac{\partial}{\partial \dot{x}_i} \left(\frac{1}{2} m \, \vec{v}^2 - V(\vec{x}) \right) \; = \; m \, \dot{x}_i \; = \; p_i \; .$$

Solution for assignment 15b: Legendre transformation.

With

$$H = \left(\sum_j \dot{q}_j \, \frac{\partial L}{\partial \dot{q}_j} - L \right)$$

we find

$$dH \;\; = \;\; \sum_j \left[d\left(\frac{\partial L}{\partial \dot{q}_j} \right) \, \dot{q}_j + \frac{\partial L}{\partial \dot{q}_j} \, d\dot{q}_j - \frac{\partial L}{\partial \dot{q}_j} \, d\dot{q}_j - \frac{\partial L}{\partial q_j} \, dq_j \right] \; .$$

The two central terms cancel out. Using Euler-Lagrange and the definition of the generalized momentum, we have

$$\frac{\partial L}{\partial q_j} \;\; = \;\; \frac{d}{dt} \frac{\partial L}{\partial \dot{q}_j} \;\; = \;\; \dot{p}_j$$

and, therefore,

$$dH \;\; = \;\; \sum_j \left(\dot{q}_j \, dp_j - \dot{p}_j \, dq_j \right) \; .$$

From this we read off Hamilton's equations:

$$\frac{\partial H}{\partial p_j} \;\; = \;\; \dot{q}_j \quad \text{and} \quad \frac{\partial H}{\partial q_j} \;\; = \;\; -\dot{p}_j \; .$$