ADVANCED DYNAMICS — PHY 4241/5227

SOLUTIONS – SET 12

Problem 45:

In units with $c = 1$ and time, distance in years: Always $t_{\rm news} = t - x$.

In the first 5 years (time unit years, distance unit light years):

$$\begin{aligned} \zeta &= \alpha\,\tau\,, \quad \alpha = g/c\,, \quad 0 \le \tau \le 5\,y \quad \text{(accelerating away from earth)}\,, \\ t &= \sinh(\zeta)/\alpha\,, \\ x &= \left[\cosh(\zeta) - 1\right]/\alpha\,. \end{aligned}$$

With $\tau_1 = 5\,y$, $\zeta_1 = \alpha\,\tau_1$, $t_1 = t(\zeta_1)$ and $x_1 = x(\zeta_1)$ we have in the next five years

$$\begin{aligned} \zeta &= \zeta_1 - \alpha\,(\tau - \tau_1)\,, \quad 5 \le \tau \le 10\,y \quad \text{(slowing down)}\,, \\ t &= t_1 + \left[t_1 - \sinh(\zeta)/\alpha\right], \\ x &= x_1 + \left\{x_1 - \left[\cosh(\zeta) - 1\right]/\alpha\right\}. \end{aligned}$$

With $\tau_2 = 2\,\tau_1$, $t_2 = 2\,t_1$ and $x_2 = 2\,x_1$ we have in the next five years

$$\begin{aligned} \zeta &= \alpha\,(\tau_2 - \tau)\,, \quad 10 \le \tau \le 15\,y \quad \text{(accelerating towards earth)}\,, \\ t &= t_2 - \sinh(\zeta)/\alpha\,, \\ x &= x_2 - \left[\cosh(\zeta) - 1\right]/\alpha\,. \end{aligned}$$

With $\tau_3 = 3\,\tau_1$, and $t_3 = 3\,t_1$ we have in the final five years ($\zeta_3 = \zeta_1,\ x_3 = x_1$)

$$\begin{aligned} \zeta &= \zeta_1 + \alpha\,(\tau - \tau_3)\,, \quad 15 \le \tau \le 20\,y \quad \text{(slowing down)}\,, \\ t &= t_3 + t_1 + \sinh(\zeta)/\alpha\,, \\ x &= x_1 - \left[\cosh(\zeta) - 1\right]/\alpha\,. \end{aligned}$$

Remark: News from the spaceship, sent of at speed of light at its time τ, reaches earth at time $t_{\text{ to earth}} = t + x$.