ADVANCED DYNAMICS — PHY 4241/5227

HOME AND CLASS WORK – SET 4

(January 26, 2011)

Read Landau-Lifshitz §10 and §11. Due January 28.

(14a) Use the Einstein summation convention in the following. Show that

$$L_i = \epsilon_{ijk}\, x_j\, p_k = [\vec{r} \times \vec{p}]_i \quad \text{holds for} \quad i = 1, 2, 3.$$

Due in class (2 points).

(14b) Express $\epsilon_{12k}\, \epsilon_{lmk}$ in terms of Kronecker delta and then $\epsilon_{ijk}\, \epsilon_{lmk}$. Due in class (2 points).

(14c) Poisson brackets are defined by

$$[g, h] = \sum_k \left(\frac{\partial g}{\partial q_k} \frac{\partial h}{\partial p_k} - \frac{\partial h}{\partial q_k} \frac{\partial g}{\partial p_k} \right)$$

where g and h are functions of q_i, p_i and, possibly, t. Show the following properties (due February 2 before class, 10 points):

1. $$\frac{dg}{dt} = [g, H] + \frac{\partial g}{\partial t} \ .$$

2. $$\dot{q}_j = [q_j, H] \ .$$

3. $$\dot{p}_j = [p_j, H] \ .$$

4. $$[x_i, x_j] = [p_i, p_j] = 0 \ ; \quad [x_i, p_j] = \delta_{ij} \ ,$$

5. $$[x_i, L_j] = \epsilon_{ijk} x_k \ , \ [p_i, L_j] = \epsilon_{ijk} p_k \ , \ \text{and} \ [L_i, L_j] = \epsilon_{ijk} L_k \ , \ ,$$

   where the Einstein summation convention is used and $L_j = \epsilon_{jkl}\, x_k\, p_l$ is the jth component of the angular momentum of the system.

Landau-Lifshitz: Skip §12, read §13 (due February 2) and §14 (due February 4).