ADVANCED DYNAMICS — PHY 4241/5227

HOME AND CLASS WORK – SET 7

(February 21, 2011)

Read Landau-Lifshitz §23. Due February 23.
Read Landau-Lifshitz §24. Due February 25.
Read Landau-Lifshitz §25. Due March 2.
Read Landau-Lifshitz §26. Due March 4.

(21a) Derive the results of problem 4 §21 of Landau and Lifshitz. Due February 23 before class (3 points).

(21b) Let $\gamma = \omega + \epsilon$. Taylor expand

$$x(t) = a\,\cos(\omega\,t + \alpha) + \frac{f\,\left[\cos(\gamma\,t + \beta) - \cos(\omega\,t + \beta)\right]}{m\,(\omega^2 - \gamma^2)}$$

to order ϵ and take the limit $\epsilon \to 0$. Due in class (3 points).

(22) Derive the results of problem 1 §22 of Landau and Lifshitz. Due February 25 before class (4 points).

(10b) Calculate the normal modes (eigenfrequencies) of the pendulum of problem 10, set 2. Due February 28 before class (4 points).

(10c) Continue with the double pendulum from assignment 10b.

1. Use the eigenfrequencies $\omega_\pm$ given in the posted solution of 10b and normal coordinates (Landau-Lifshitz p.67/8) to write down the general solution for the two angles.
2. Express the integration constants of your solution through the angular positions and velocities at time $t = 0$, denoted by ϕ_0, $\dot{\phi}_0$, ψ_0, $\dot{\psi}_0$.
3. Use $\sqrt{l/g}$ as time unit and plot the solutions $\phi(t)$ and $\psi(t)$ up to $t = 50\sqrt{l/g}$ for initial conditions $\phi_0 = 0,\ \dot{\phi}_0 = 1,\ \psi_0 = 0,\ \dot{\psi}_0 = -1$.

Due March 4 before class (10 points).

(21c) Consider $x(t) = a\,\exp(-\lambda\,t)\,\cos(\omega\,t + \alpha)$. Let $\omega = \epsilon$, $a = a_1/\epsilon$, $\alpha = \pi/2 + \epsilon/a_2$, $a_1,\ a_2 \neq 0$ and calculate $x(t)$ in the limit $\epsilon \to 0$. Due March 14 before class (3 points).