ADVANCED DYNAMICS — PHY 4241/5227 HOME AND CLASS WORK – SET 9

(March 29, 2010)

- (26) Exercises with the rapidity variable.
 - 1. Show $1/\sqrt{1-\beta^2} = \cosh(\zeta)$. Due in class (2 points).
 - 2. Show $\cosh(\zeta_2) \cosh(\zeta_1) + \sinh(\zeta_2) \sinh(\zeta_1) = \cosh(\zeta_1 + \zeta_2)$. Due in class (2 points).
 - 3. Velocities are given by $\beta_1 = \tanh(\zeta_1)$ and $\beta_2 = \tanh(\zeta_2)$. Use the addition theorem of rapidities to express the velocity given by $\beta = \tanh(\zeta_1 + \zeta_2)$ in terms of β_1 and β_2 . Due in class (4 points).
- (27) D mesons have a (mean) lifetime of 1051×10^{-15} s. Assume such a meson is created in a particle collission at Fermilab and moves at 0.995 c with respect to the Lab frame. Compute the (mean) distance travelled by such a D meson. Due April 4 before class (6 points).
- (28) Assume that a rocket ship leaves the earth in the year 2020. The rocket ship is so constructed so that it has an acceleration g in its own frame (to make the occupants feel comfortable). It accelerates on a straight-line path for 5 years (by its own clocks), decelerates at the same rate for 5 more years, turns around, accelerates for 5 years, decelerates for 5 years, and lands on earth. What year is on earth? Use $g = 9.81 \ [m/s^2]$, one year $= 365 \times 24 \times 3600 \ [s]$, and the speed of light c, and assume that earth defines an inertial system. Due April 4 before class (10 points).
- (29) Exercises with tensors.
 - 1. Let $S_{\alpha\beta}$ be a (covariant) symmetric tensor. Write $S_{\alpha\beta} dx^{\alpha} dx^{\beta}$ explicitly as a sum of ten terms. Due in class (2 points).
 - 2. Let $F_{\alpha\beta}$ be a (covariant) antisymmetric tensor. Calculate $F_{\alpha\beta} dx^{\alpha} dx^{\beta}$. Due in class (2 points).
 - 3. Calculate $h^{\alpha}_{\ \beta} = g^{\alpha\gamma}g_{\gamma\beta}$ and $r^{\ \beta}_{\alpha} = g_{\alpha\gamma}g^{\gamma\beta}$. (A) As *g*-tensor. (B) The explicit matrix and identify then with another well-known symbol. Due in class (2 points).
 - 4. Write down explicitly the matrix (a^{β}_{α}) and its transpose $(\tilde{a}^{\alpha}_{\beta}), \tilde{a}^{\alpha}_{\beta} = a^{\beta}_{\alpha}$. Due in class (2 points).
- (30) Continuation of (28). Seen from earth, how far away did the spaceship travel? Express the result in light years. Due April 6 before class (6 points).