## ADVANCED DYNAMICS — PHY 4241/5227 HOME AND CLASS WORK – SET 11

(April 13, 2011)

- (34) For light from some galaxy the Balmer spectrum of the hydrogen atom is found at wavelengths  $\lambda = (729.2 \text{ nm}) m^2/(m^2-4), m = 3, 4, 5, \ldots$ . Neglect expansion of space and find the speed at which the galaxy moves away or towards us. Due April 15 before class (4 points).
- (35a) Derive for the momentum in the rest frame  $p^0 = c m_0$ , where  $m_0$  is the rest mass, from the relativistic transformation laws and the known definition of momentum in the non-relativistic limit. Due in class (NOT done).
- (35b) (1) Taylor expand  $\sqrt{x+\epsilon}$  to leading order in  $\epsilon$ . (2) Substitute  $x = c^4 m_0^2$  and  $\epsilon = c^2 \vec{p}^2$ . (3) Compare with  $mv^2/2$ . Due in class (3 points).
- (36) End of the spaceship journey: Assume that the spaceship moves by transforming mass into light, which is exhausted.
  - 1. Derive an expression for  $m(\tau)$ , the (remaining) mass of the spaceship at proper time  $\tau$ . (Hint: Use momentum conservation in the instanteneous rest frame.)
  - 2. Which fraction of the original mass is left, after the spacetrip has been performed?

Due April 21 before class (8 points).

(37) (1) Write down the definition of  $\partial_{\alpha}$ . Due in class (1 point).

(2) Let  $A^{\alpha}$  be a vector field. Write down the scalar field from this and  $\partial_{\alpha}$  (not the scalar operator). Due in class (1 point).

(3) Use the quantities of assignments (1) and (2) to write down a rank two tensor  $T^{\alpha\beta}$  field (not a tensor operator). Due in class (1 point).

(4) Write the equation

$$\partial_{\alpha} F^{\alpha\beta} = \frac{4\pi}{c} J^{\beta}$$

explicitly out for  $\beta = 0, 1, 2, 3$ . Due in class (2 points).

(5) Compare the  $\beta = 0$  result with  $\nabla \cdot \vec{E} = 4\pi\rho$  and the  $\beta = i = 1, 2, 3$  results with  $\nabla \times \vec{B} - c^{-1} \left( \partial \vec{E} / \partial t \right) = 4\pi \vec{J} / c$ . Due in class (2 points).