ADVANCED DYNAMICS — PHY–4241/5227

Midterm Exam March 21, 2011

Each problem counts 25 points.

PROBLEM 1

Write the principle of least action and the Euler-Lagrange equation(s) of motion for a 1-dimensional Lagrangian of the form:

$$L = \frac{1}{2} m \dot{x}^2 - V(x) \ .$$

Is the resulting equation consistent with Newton's second law?

PROBLEM 2

The Lagrangian of the 1D harmonic oscillator is

$$L = \frac{1}{2}\, m\, \dot{x}^2 - \frac{1}{2}\, k\, x^2 \ .$$

1. Use the definition of the generalized momentum to find the momentum p.
2. Write down the Hamiltonian $H(p, x)$.
3. Write down Hamilton's equations.
4. Show that Hamilton's equations give Newton's force law.

PROBLEM 3

Consider the 3-dimensional Lagrangian for a particle of mass m moving in the presence of a spherically symmetric potential $V(\mathbf{r}) = V(r)$. That is,

$$L = \frac{1}{2} m \Big(\dot{r}^2 + r^2 \dot{\theta}^2 + r^2 \sin^2\theta \dot{\phi}^2 \Big) - V(r) \ .$$

Identify as many conserved quantities (*i.e.,* constants of the motion) as you can from the mere structure of the Lagrangian.

PROBLEM 4

Consider a point mass m on the surface of a sphere of radius R under the influence of gravity $-g\hat{z}$ (spherical pendulum).

1. Write down the Lagrange function using spherical coordinates.
2. Find the Euler-Lagrange equations.
3. Calculate the special solutions for $\theta =$ constant. Describe this motion.