ADVANCED DYNAMICS — PHY–4241/5227

Midterm Exam Solutions

PROBLEM 1

From all possible paths that a physical system can take in going from point x_1 at time t_1 to point x_2 at time t_2 it will select the one that minimizes the action

$$S = \int_{t_1}^{t_2} dt\, L = \int_{t_1}^{t_2} dt\, (T - V)$$

under local variations of the path. The Euler-Lagrange equations follow. In our case:

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{x}} = \frac{\partial L}{\partial x} \;\Rightarrow\; m\ddot{x} = -V'(x) = F(x)$$

in agreement with Newton's second law.

PROBLEM 2

(1) The momentum is

$$p = \frac{\partial L}{\partial \dot{x}} = m\dot{x}$$

(2) Eliminating $\dot{x}$ in favor of p we obtain the Hamiltonian

$$H = T + V = \frac{p^2}{2m} + \frac{1}{2}kx^2$$

(3) Hamilton's equations are

$$\frac{\partial H}{\partial p} = \frac{p}{m} = \dot{x} \;\text{ and }\; \frac{\partial H}{\partial x} = kx = -\dot{p}$$

(4) Newton's force law follows

$$kx = -\dot{p} = -m\ddot{x} \;\text{ or }\; m\ddot{x} = -kx$$

PROBLEM 3

The given Lagrangian is independent of t and ϕ. Thus,

$$\frac{\partial L}{\partial t} = 0 \;\Rightarrow\; E = T + V \;\text{ conserved.}$$

and

$$0 = \frac{\partial L}{\partial \phi} = \frac{d}{dt}\frac{\partial L}{\partial \dot{\phi}} = mr^2 \sin^2(\theta)\dot{\phi} = p_\phi \;\text{ conserved.}$$