FINAL ADVANCED DYNAMICS – PHY 4241/5227 (April 27, 2011)

PROBLEM 1 (25 points)

Assume a Lagrangian $L = L(\{q_i\}, \{\dot{q}_i\}, t)$ where $q_i, i = 1, ..., n$ are generalized coordinates, $\dot{q}_i, i = 1, ..., n$ are generalized velocities and t is the time.

- 1. Write down the principle of least action.
- 2. Derive the Euler-Lagrange equations from the principle of least action.
- 3. Assume that the Lagrangian is invariant under translations $q_i \rightarrow q'_i = q_i + \epsilon_i$. Find the corresponding conserved quantities.

PROBLEM 2 (25 points)

- 1. Write down the two basic postulates of the Special Theory of Relativity.
- 2. The lifetime of a pion is $\tau = 2.6 \times 10^{-8}$ s (with respect to its rest frame). Assume the pion travels in a collider experiment at 99% of the speed of light with respect to the Lab frame.
 - (a) How long is the observed lifetime of the pion in the Lab frame?
 - (b) How far does the pion travel in the Lab frame during one lifetime?

PROBLEM 3 (25 points)

The electromagnetic field tensor is given by

$$(F^{\alpha\beta}) = \begin{pmatrix} 0 & -E^x & -E^y & -E^z \\ E^x & 0 & -B^z & B^y \\ E^y & B^z & 0 & -B^x \\ E^z & -B^y & B^x & 0 \end{pmatrix} \ .$$

Find (the Einstein summation convention is assumed for identical indices):

- 1. $F^{\alpha}{}_{\alpha}$.
- 2. The matrix $(F_{\alpha\beta})$.
- 3. $F_{\alpha\beta}F^{\alpha\beta}$.

PROBLEM 4 (25 points)

An electron and a positron, each with mass equal to 0.511 MeV, annihilate at rest into two photons.

- 1. Chose the rest frame of the positronium, such that one of the photons moves in positive $x = x^1$ direction. For each photon find the momentum 4-vector.
- 2. Find for one of the photons the 4-momentum in a frame that moves at a velocity $c\beta \hat{x}$ along the previously defined x^1 direction with respect to the rest frame of the positronium.
- 3. Suppose that the annihilation took place 10^9 years ago in a galaxy that is receding from us at $\beta = 4/5$. What is the energy of the photon that we observe?