FINAL ADVANCED DYNAMICS – PHY 4241/5227 (April 27, 2011)

PROBLEM 1 (25 points)

Assume a Lagrangian $L = L(\{q_i\}, \{\dot{q}_i\}, t)$ where q_i, $i = 1, \ldots, n$ are generalized coordinates, $\dot{q}_i$, $i = 1, \ldots, n$ are generalized velocities and t is the time.

1. Write down the principle of least action.
2. Derive the Euler-Lagrange equations from the principle of least action.
3. Assume that the Lagrangian is invariant under translations $q_i \to q_i' = q_i + \epsilon_i$. Find the corresponding conserved quantities.

PROBLEM 2 (25 points)

1. Write down the two basic postulates of the Special Theory of Relativity.
2. The lifetime of a pion is $\tau = 2.6 \times 10^{-8}$ s (with respect to its rest frame). Assume the pion travels in a collider experiment at 99% of the speed of light with respect to the Lab frame.
   (a) How long is the observed lifetime of the pion in the Lab frame?
   (b) How far does the pion travel in the Lab frame during one lifetime?

PROBLEM 3 (25 points)

The electromagnetic field tensor is given by

$$(F^{\alpha\beta}) = \begin{pmatrix} 0 & -E^x & -E^y & -E^z \\ E^x & 0 & -B^z & B^y \\ E^y & B^z & 0 & -B^x \\ E^z & -B^y & B^x & 0 \end{pmatrix} .$$

Find (the Einstein summation convention is assumed for identical indices):

1. ${F^\alpha}_\alpha$.
2. The matrix $(F_{\alpha\beta})$.
3. $F_{\alpha\beta}F^{\alpha\beta}$.

PROBLEM 4 (25 points)

An electron and a positron, each with mass equal to 0.511 MeV, annihilate at rest into two photons.

1. Chose the rest frame of the positronium, such that one of the photons moves in positive $x = x^1$ direction. For each photon find the momentum 4-vector.
2. Find for one of the photons the 4-momentum in a frame that moves at a velocity $c\beta\hat{x}$ along the previously defined x^1 direction with respect to the rest frame of the positronium.
3. Suppose that the annihilation took place 10^9 years ago in a galaxy that is receding from us at $\beta = 4/5$. What is the energy of the photon that we observe?