SOLUTIONS FINAL ADVANCED DYNAMICS

PROBLEM 4

Total energy: $p^0 + q^0 = 2 E$, E = 0.511 MeV. Momentum conservation: $\vec{p} = -\vec{q}$. Zero photon rest mass: $p_{\alpha}p^{\alpha} = q_{\alpha}q^{\alpha} = 0$.

1. CM frame (rest from of the positronium, c = 1):

$$(p^{\alpha}) = \begin{pmatrix} E \\ E \\ 0 \\ 0 \end{pmatrix}, \quad (q^{\alpha}) = \begin{pmatrix} E \\ -E \\ 0 \\ 0 \end{pmatrix}.$$

2. The transformation to the moving frame is

$$(p'^{\alpha}) = \begin{pmatrix} \gamma & -\gamma \beta & 0 & 0\\ -\gamma \beta & \gamma & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1 \end{pmatrix} (p^{\alpha}) = \begin{pmatrix} \gamma E - \gamma \beta E\\ -\gamma E + \gamma \beta E\\ 0\\ 0 \end{pmatrix}.$$

3. The observed energy follows from $\beta = 4/5$ and $\gamma = 1/\sqrt{1-\beta^2} = 5/3$ to be $E' = \gamma (1-\beta) E = E/3 = 0.17 \,\text{MeV}.$