$$\vec{F} = m\dot{\vec{v}} = \frac{q}{c}\vec{V}\times\vec{B} = \left(\frac{qB_0}{c}\right)\vec{V}\times\hat{z}$$

OR

$$\frac{d\vec{V}}{dt} = \left(\frac{qB_0}{mc}\right)\vec{V}\times\hat{z} = \Omega\left[\hat{x}V_x + \hat{y}V_y + \hat{z}V_z\right]\times\hat{z}$$

$$= \Omega\left(\hat{x}V_y - \hat{y}V_x\right);\quad \Omega \equiv \frac{qB_0}{mc}$$

OR

$$\frac{dV_x}{dt} = \Omega V_y;\quad \frac{dV_y}{dt} = -\Omega V_x;\quad \frac{dV_z}{dt} = 0$$

HENCE,

$$V_z(t) = V_z(0) = \text{CONSTANT}$$

AND

$$\frac{d^2V_x}{dt^2} = \Omega\frac{dV_y}{dt} = \Omega\left[-\Omega V_x\right] \Rightarrow \ddot{V}_x + \Omega^2 V_x = 0$$

THUS,

$$V_x(t) = A\cos\Omega t + B\sin\Omega t$$

AND

$$V_y(t) = \frac{1}{\Omega}\frac{dV_x}{dt} = -A\sin\Omega t + B\cos\Omega t$$

OR

$$\boxed{\begin{aligned} V_x(t) &= V_x(0)\cos\Omega t + V_y(0)\sin\Omega t \\ V_y(t) &= V_y(0)\cos\Omega t - V_x(0)\sin\Omega t \\ V_z(t) &= V_z(0);\quad \Omega \equiv qB_0/mc \end{aligned}}$$