

ADVANCED DYNAMICS — PHY 4241/5227
SOLUTIONS – SET 9

(26.1)

$$\frac{1}{\sqrt{1 - \tanh^2(\zeta)}} = \sqrt{\frac{\cosh^2(\zeta)}{\cosh^2(\zeta) - \sinh^2(\zeta)}} = \cosh(\zeta) .$$

(26.2)

$$\begin{aligned} & \cosh(\zeta_1) \cosh(\zeta_2) + \sinh(\zeta_1) \sinh(\zeta_2) \\ &= \frac{e^{\zeta_1} + e^{-\zeta_1}}{2} \frac{e^{\zeta_2} + e^{-\zeta_2}}{2} + \frac{e^{\zeta_1} - e^{-\zeta_1}}{2} \frac{e^{\zeta_2} - e^{-\zeta_2}}{2} \\ &= \frac{e^{\zeta_1 + \zeta_2} + 2e^{\zeta_1 - \zeta_2} + e^{-\zeta_1 - \zeta_2}}{4} + \frac{e^{\zeta_1 + \zeta_2} - 2e^{\zeta_1 - \zeta_2} + e^{-\zeta_1 - \zeta_2}}{4} \\ &= \cosh(\zeta_1 + \zeta_2) \end{aligned}$$

(26.3)

$$\begin{aligned} \tanh(\zeta_1 + \zeta_2) &= \frac{\sinh(\zeta_1 + \zeta_2)}{\cosh(\zeta_1 + \zeta_2)} \\ &= \frac{\sinh(\zeta_1) \cosh(\zeta_2) + \cosh(\zeta_1) \sinh(\zeta_2)}{\cosh(\zeta_1) \cosh(\zeta_2) + \sinh(\zeta_1) \sinh(\zeta_2)} \\ &= \frac{\tanh(\zeta_1) + \tanh(\zeta_2)}{1 + \tanh(\zeta_1) \tanh(\zeta_2)} = \frac{\beta_1 + \beta_2}{1 + \beta_1 \beta_2} \end{aligned}$$