Problem 39: Invariance under Gauge Transformations.

$$F^{\alpha\beta} = \partial^\alpha A^\beta - \partial^\beta A^\alpha \,, \quad A'^{\,\alpha} = A^\alpha + \partial^\alpha \Lambda \,.$$

Then,

$$F'^{\,\alpha\beta} = \partial^\alpha A'^{\,\beta} - \partial^\beta A'^{\,\alpha} = \partial^\alpha A^\beta + \partial^\alpha \partial^\beta \Lambda - \partial^\beta A^\alpha - \partial^\beta \partial^\alpha \Lambda \,. = \partial^\alpha A^\beta - \partial^\beta A^\alpha = F^{\alpha\beta} \,.$$