ADVANCED DYNAMICS — PHY 4241/5227

HOME AND CLASS WORK – SET 3

(January 20, 2010)

(13) A double pendulum consists of two simple pendula, with one pendulum suspended from the bob of the other. Assume that the two pendula have equal lengths, have bobs of equal mass and are confined to move in the same plane. (Compare problem 1 of Landau-Lifshitz with $l_1 = l_2 = l$, $\phi_1 = \phi$ and $\phi_2 = \psi$.)

1. Define angles ϕ and ψ for the pendula with respect to the gravity direction and write down the Lagrange function.
2. Derive the equations of motion for small oscillations around the rest position $\phi = \psi = 0$. (Small oscillations neglect all terms in the Taylor expansion of the Lagrangian, which are higher than quadratic in combinations of ϕ, $\dot{\phi}$, ψ, $\dot{\psi}$.)
3. Calculate the normal modes.

Due January 25 before class (10 points).

Read M&T chapter 7.9 (due January 25).

(14) Derive (due January 25 in class, 8 points)

$$\frac{d}{dt}\left(L - \sum_j \dot{q}_j \frac{\partial L}{\partial \dot{q}_j}\right) = 0 \ \text{ from } \ \delta_t L = \frac{\partial L}{\partial t}\,\delta t = 0 \ .$$

Assume a bilinear kinetic Energy

$$T = \sum_{j,k} a_{jk}\,\dot{q}_j \dot{q}_k \ \text{ and prove } \ \sum_i \dot{q}_i \frac{\partial T}{\partial \dot{q}_i} = 2T \ .$$

(15a) Generalized Momentum:

$$\text{Calculate } \frac{\partial L}{\partial x_i},\ i = 1,2,3,\ \text{for } L = \frac{1}{2}\,m\,\vec{v}^{\,2} - V(\vec{x}).$$

Due January 29 in class (2 points).

(15b) Legendre transformation: Define the Hamiltonian by

$$H = \left(\sum_j \dot{q}_j \frac{\partial L}{\partial \dot{q}_j} - L\right) \text{ and the generalized momentum by } p_j = \frac{\partial L}{\partial \dot{q}_j} \ .$$

Show that the Hamiltonian is a function of q_j and p_j only: $H = H(q_j, p_j)$. Then derive Hamilton's equations of motion. Hint: Calculate dH. Due January 29 in class (6 points).