ADVANCED DYNAMICS — PHY 4241/5227

HOME AND CLASS WORK – SET 2

(January 7, 2011)

(6) Expand to order ϵ (each one point):

1. $f(x+\epsilon)$.
2. $f(x_1+\epsilon_1,\dots,x_n+\epsilon_n)$.
3. $f(v_1+\epsilon_1,\dots,v_n+\epsilon_n)$.
4. $v'^2 = v^2(v_1+\epsilon_1, v_2+\epsilon_2, v_3+\epsilon_3)$ with $v^2 = v^2(v_1,v_2,v_3)$.
5. $f(v'^2)$.

Due January 10 before class (up to 5 points).

Assignments 7 and 8 are motivated by Feynman's text:

(7) Show that "the mean square of something that deviates around an average ... is always greater that the square of the mean" (4 points). Due in class (4 points).

(8) What is a 3D conservative force? Write down two definitions (2 points). Due in class (2 points).

(9) Derive the Euler-Lagrange equations for

$$\delta \int_{t_1}^{t_2} dt\, L = 0\,, \qquad L = L(q_i,\dot{q}_i,t)\,, \qquad i = 1,\dots,s\,.$$

Due in class (8 points).

– Read Landau-Lifshitz §6, §7, §8. Due January 14 before class.

(10) A double pendulum consists of two simple pendula, with one pendulum suspended from the bob of the other. Assume that the two pendula have equal lengths, have bobs of equal mass and are confined to move in the same plane. (Compare problem 1 of Landau-Lifshitz with $l_1 = l_2 = l$, $\phi_1 = \phi$ and $\phi_2 = \psi$.)

1. Define angles ϕ and ψ for the pendula with respect to the gravity direction and write down the Lagrange function.
2. Derive the equations of motion for small oscillations around the rest position $\phi = \psi = 0$. (Small oscillations neglect all terms in the Taylor expansion of the Lagrangian, which are higher than quadratic in combinations of ϕ, $\dot{\phi}$, ψ, $\dot{\psi}$.)

Due January 19 before class (10 points).