ADVANCED DYNAMICS — PHY 4241/5227

HOME AND CLASS WORK – SET 3

(January 21, 2011)

(11a) Assume a bilinear kinetic Energy

$$T = \sum_{j,k} a_{jk}\, \dot{q}_j \dot{q}_k \quad \text{and prove} \quad \sum_i \dot{q}_i\, \frac{\partial T}{\partial \dot{q}_i} = 2T\ .$$

Due in class (2 points).

(11b) Generalized Momentum:

$$\text{Calculate } \frac{\partial L}{\partial \dot{x}_i},\ i = 1, 2, 3,\ \text{for } L = \frac{1}{2}\, m\, \vec{v}^{\,2} - V(\vec{x}).$$

Due in class (2 points).

(12) Legendre transformation: Define the Hamiltonian by

$$H = \left(\sum_j \dot{q}_j\, \frac{\partial L}{\partial \dot{q}_j} - L \right) \text{ and the generalized momentum by } p_j = \frac{\partial L}{\partial \dot{q}_j}\ .$$

Show that the Hamiltonian is a function of q_j and p_j only: $H = H(q_j, p_j)$. Then derive Hamilton's equations of motion. Hint: Calculate dH. Due January 26 before class (10 points).