ADVANCED DYNAMICS — PHY 4241/5227

HOME AND CLASS WORK – SET 5

(February 4, 2011)

	Masses		Initial Positions			Initial Velocities		
#	i	m_i	$x^1_{i,0}$	$x^2_{i,0}$	$x^3_{i,0}$	$\dot{x}^1_{i,0}$	$\dot{x}^2_{i,0}$	$\dot{x}^3_{i,0}$
1	1	0.651	0.585	-0.238	-0.755	-0.828	-0.865	-0.726
	2	0.931	-0.096	0.000	0.357	-0.209	0.107	-0.660
2	1	1.510	0.460	-0.359	-0.234	-0.918	-0.941	-0.323
	2	0.126	-0.066	-0.090	-0.809	0.789	0.788	0.620
3	1	1.328	-0.125	0.898	0.194	-0.452	0.172	0.125
	2	1.999	-0.449	-0.085	-0.454	-0.976	-0.990	-0.968
4	1	0.180	0.204	-0.968	-0.753	-0.811	-0.632	0.784
	2	1.560	-0.889	-0.979	0.854	-0.323	-0.774	-0.533

Table 1: Initial conditions for the Kepler problem (arbitrary units and set $G = 1$).

(15) Calculate the angular momentum vectors in the CM frame for the initial conditions given in the table above (you can download the data as text file Kepler.txt). Due February 7 before class 8 points.

(16) Let the interaction of two point particles be described by a potential which depends only on their distance:

$$\mathcal{L} = \frac{m_1}{2}\vec{v}_1^{\,2} + \frac{m_2}{2}\vec{v}_2^{\,2} - U(r)\,, \quad r = |\vec{r}|\,,\ \vec{r} = \vec{r}_1 - \vec{r}_2\ .$$

1. Is the energy of this system conserved (with reason) (1 point)?
2. Define the center of mass vector by $\vec{R} = (m_1\vec{r}_1 + m_2\vec{r}_2)/M$, $M = m_1 + m_2$ and express $\vec{r}_1$ and $\vec{r}_2$ through $\vec{R}$ and $\vec{r}$ (2 points).

   The center of mass (cm) frame is defined by $\vec{R}(t) = 0$. Show the following equalities in the cm system:

3. $T_{\rm cm} = m_1\,\vec{v}_1^{\,2}/2 + m_2\,\vec{v}_2^{\,2}/2 = \mu\,\vec{v}^{\,2}/2$ with $\vec{v} = \dot{\vec{r}}$ and μ the reduced mass. Express μ through m_1, m_2 and M (2 points).
4. $\vec{L} = \vec{r}_1 \times \vec{p}_1 + \vec{r}_2 \times \vec{p}_2 = \mu\,\vec{r} \times \vec{v}$ (2 points).

Due in class.

(17) Plot the effective potentials corresponding to the initial conditions of the table together with the CM energies (the Potential is $U(r) = -G\,m_1 m_2/r$). Due February 11 before class 12 points.