

ADVANCED DYNAMICS — PHY 4241/5227
HOME AND CLASS WORK – SET 3

Solution for assignment 12:

With

$$H = \left(\sum_j \dot{q}_j \frac{\partial L}{\partial \dot{q}_j} - L \right)$$

we find

$$dH = \sum_j \left[d \left(\frac{\partial L}{\partial \dot{q}_j} \right) \dot{q}_j + \frac{\partial L}{\partial \dot{q}_j} d\dot{q}_j - \frac{\partial L}{\partial \dot{q}_j} d\dot{q}_j - \frac{\partial L}{\partial q_j} dq_j \right] .$$

The two central terms cancel out. Using Euler-Lagrange and the definition of the generalized momentum, we have

$$\frac{\partial L}{\partial q_j} = \frac{d}{dt} \frac{\partial L}{\partial \dot{q}_j} = \dot{p}_j$$

and, therefore,

$$dH = \sum_j (\dot{q}_j dp_j - \dot{p}_j dq_j) .$$

From this we read off Hamilton's equations:

$$\frac{\partial H}{\partial p_j} = \dot{q}_j \quad \text{and} \quad \frac{\partial H}{\partial q_j} = -\dot{p}_j .$$