ADVANCED DYNAMICS — PHY 4241/5227 HOME AND CLASS WORK – SET 3

(January 21, 2011)

(11a) Assume a bilinear kinetic Energy

$$T = \sum_{j,k} a_{jk} \, \dot{q}_j \dot{q}_k$$
 and prove $\sum_i \dot{q}_i \, \frac{\partial T}{\partial \dot{q}_i} = 2T$.

Due in class (2 points).

(11b) Generalized Momentum:

Calculate
$$\frac{\partial L}{\partial \dot{x}_i}$$
, $i = 1, 2, 3$, for $L = \frac{1}{2} m \vec{v}^2 - V(\vec{x})$.

Due in class (2 points).

(12) Legendre transformation: Define the Hamiltonian by

$$H = \left(\sum_{j} \dot{q}_{j} \frac{\partial L}{\partial \dot{q}_{j}} - L\right) \text{ and the generalized momentum by } p_{j} = \frac{\partial L}{\partial \dot{q}_{j}}$$

Show that the Hamiltonian is a function of q_j and p_j only: $H = H(q_j, p_j)$. Then derive Hamilton's equations of motion. Hint: Calculate dH. Due January 26 before class (10 points).