ADVANCED DYNAMICS — PHY 4241/5227

HOME AND CLASS WORK – SET 3

(January 24, 2011)

(11a) Assume a bilinear kinetic Energy

$$T = \sum_{j,k} a_{jk}\, \dot{q}_j \dot{q}_k \quad \text{and prove} \quad \sum_i \dot{q}_i \frac{\partial T}{\partial \dot{q}_i} = 2T \ .$$

Due in class (2 points).

(11b) Generalized Momentum:

$$\text{Calculate } \frac{\partial L}{\partial \dot{x}_i},\ \ i = 1, 2, 3,\ \text{ for } L = \frac{1}{2}\, m\, \vec{v}^{\,2} - V(\vec{x}).$$

Due in class (2 points).

(12) Legendre transformation: Define the Hamiltonian by

$$H = \left(\sum_j \dot{q}_j \frac{\partial L}{\partial \dot{q}_j} - L \right) \text{ and the generalized momentum by } p_j = \frac{\partial L}{\partial \dot{q}_j} \ .$$

Show that the Hamiltonian is a function of q_j and p_j only: $H = H(q_j, p_j)$. Then derive Hamilton's equations of motion. Hint: Calculate dH. Due January 26 before class (10 points).

(13a) Calculate explicitly

$\delta_x L$ for the Lagrangian of two harmonically bound particles in 1D

$$L = \frac{m_1}{2} (\dot{x}_1)^2 + \frac{m_2}{2} (\dot{x}_2)^2 - k\, (x_1 - x_2)^2 \ .$$

Is there a conservation law? Due in class (2 points).

(13a) Repeat the previous excercise for the harmonic ocillator in 1D

$$L = \frac{m_1}{2} (\dot{x})^2 - k\, x^2 \ .$$