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PHY4241

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11b)

$$L = \frac{1}{2}m\vec{v^2} - V(\vec{x}) \tag{1}$$

$$\vec{v^2} = \dot{x_1}^2 + \dot{x_2}^2 + \dot{x_3}^2 \tag{2}$$

$$\frac{\partial L}{\partial \dot{x_i}} = \frac{\partial L}{\partial \dot{x_1}}\hat{x_1} + \frac{\partial L}{\partial \dot{x_2}}\hat{x_2} + \frac{\partial L}{\partial \dot{x_3}}\hat{x_3} \tag{3}$$

$$\begin{aligned}
\frac{\partial L}{\partial \dot{x_i}} &= \frac{1}{2}m\frac{\partial \vec{v^2}}{\partial \dot{x_i}} \\
&= \frac{1}{2}m(2\dot{x_1}\hat{x_1} + 2\dot{x_2}\hat{x_2} + 2\dot{x_3}\hat{x_3}) \\
&= m\dot{x_1}\hat{x_1} + m\dot{x_2}\hat{x_2} + m\dot{x_3}\hat{x_3} \\
&= p_1\hat{x_1} + p_2\hat{x_2} + p_3\hat{x_3} \\
&= p_i
\end{aligned} \tag{4}$$

13a)

$$L = \frac{m_1}{2}(\dot{x_1})^2 + \frac{m_2}{2}(\dot{x_2})^2 - k(x_1 - x_2)^2 \tag{5}$$

$$\begin{aligned}
\delta_x L &= \frac{\partial L}{\partial x_1}\delta x_1 + \frac{\partial L}{\partial x_2}\delta x_2 \\
&= -2k(x_1 - x_2)\delta x_1 + 2k(x_1 - x_2)\delta x_2 \\
&= -2k(x_1 - x_2)\delta x + 2k(x_1 - x_2)\delta x \\
&= 0
\end{aligned} \tag{6}$$

where $\delta x_1 = \delta x_2 = \delta x$ because both particles are being translated by the same amount.

This shows that the sum of the forces in a closed system is 0. The force particle 1 exerts on particle 2 is equal and opposite to the force particle 2 exerts on particle 1.

13b)

$$L = \frac{m_1}{2}\dot{x}^2 - kx^2 \tag{7}$$

$$\begin{aligned}
\delta_x L &= \frac{\partial L}{\partial x}\delta x \\
&= -2kx\delta x
\end{aligned} \tag{8}$$

This is not a closed system so the sum of the forces is not 0.