Fluctuation about mean velocity:

$$\bar{v} = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} v(t)\, dt$$

$$0 \leq \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} (v(t) - \bar{v})^2 \, dt$$

$$= \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} \left[v(t)^2 - 2\bar{v}\, v(t) + \bar{v}^2 \right] dt$$

$$= \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} v(t)^2 \, dt - \frac{2\bar{v}}{t_2 - t_1} \int_{t_1}^{t_2} v(t)\, dt + \bar{v}^2$$

$$= \overline{v^2} - 2\bar{v}^2 + \bar{v}^2 = \underline{\underline{\overline{v^2} - \bar{v}^2}}$$

Conservative force definitions:

1. $\oint \vec{F} \cdot d\vec{s} = 0$ or $\int_{\vec{r}_1}^{\vec{r}_2} \vec{F} \cdot d\vec{s}$ path independent

2. $\nabla \times \vec{F} = 0$

3. $\vec{F} = -\nabla V(\vec{r})$