ADVANCED DYNAMICS — PHY 4241/5227

HOME AND CLASS WORK – SET 2

Solution for assignment 9.

Deviation of the Euler-Lagrange equations from the least action principle in general coordinates $q_i, \dot{q}_i$, $i = 1, \dots n$:

$$
\begin{aligned}
0 &= \delta \int_{t_1}^{t_2} dt\, L(q_i, \dot{q}_i, t) \;=\; \int_{t_1}^{t_2} dt\, \{L(q_i + \delta q_i, \dot{q}_i + \delta \dot{q}_i, t) - L(q_i, \dot{q}_i, t)\} \\
&= \int_{t_1}^{t_2} dt \left\{ L(q_i, \dot{q}_i, t) + \sum_i \frac{\partial L}{\partial q_i}\, \delta q_i + \sum_i \frac{\partial L}{\partial \dot{q}_i}\, \delta \dot{q}_i - L(q_i, \dot{q}_i, t) \right\} \\
&= \int_{t_1}^{t_2} dt \left\{ \sum_i \frac{\partial L}{\partial q_i}\, \delta q_i + \sum_i \frac{\partial L}{\partial \dot{q}_i}\, \frac{d}{dt} \delta q_i \right\} \\
&= \int_{t_1}^{t_2} dt \sum_i \left\{ \frac{\partial L}{\partial q_i} - \frac{d}{dt} \frac{\partial L}{\partial \dot{q}_i} \right\} \delta q_i + \left[\frac{\partial L}{\partial \dot{q}_i}\, \delta q_i \right]_{t_1}^{t_2} \;=\; \int_{t_1}^{t_2} dt \sum_i \left\{ \frac{\partial L}{\partial q_i} - \frac{d}{dt} \frac{\partial L}{\partial \dot{q}_i} \right\} \delta q_i \;.
\end{aligned}
$$

The last equality holds because of $\delta q_i(t_1) = \delta q_i(t_2) = 0$. As the variations are independent, the finally obtained relation is equivalent to

$$
\frac{\partial L}{\partial q_i} - \frac{d}{dt} \frac{\partial L}{\partial \dot{q}_i} \;=\; 0 \;\; \text{for} \;\; i = 1, \dots, n \;.
$$