ADVANCED DYNAMICS — PHY 4241/5227

HOME AND CLASS WORK – SET 3

(January 20, 2010)

Solution for assignment 11a:

For a bilinear kinetic Energy we find by differentiation and carrying the sum over i out

$$\begin{aligned}\sum_i \dot{q}_i \frac{\partial T}{\partial \dot{q}_i} &= \sum_i \dot{q}_i \frac{\partial}{\partial \dot{q}_i} \sum_{j,k} a_{jk}\, \dot{q}_j \dot{q}_k \;=\; \sum_i \dot{q}_i \sum_{j,k} a_{jk}\, (\delta_{ij}\, \dot{q}_k + \dot{q}_j\, \delta_{ik}) \\ &= \sum_{j,k} a_{jk}\, (\dot{q}_j\, \dot{q}_k + \dot{q}_j\, \dot{q}_k) \;=\; 2\, T\ .\end{aligned}$$

Solution for assignment 11b: Motivation of generalized momentum definition.

$$\frac{\partial}{\partial \dot{x}_i} \left(\frac{1}{2}\, m\, \vec{v}^2 - V(\vec{x}) \right) \;=\; m\, \dot{x}_i \;=\; p_i\ .$$