

ADVANCED DYNAMICS — PHY 4936

HOME AND CLASS WORK – SET 5

(October 13, 2011)

Read Landau-Lifshitz up to p.39 (§15).

- (19) Consider the effective potential of the Kepler problem

$$U_{\text{eff}}(r) = -\frac{\alpha}{r} + \frac{L^2}{2mr^2} .$$

- (A) Calculate the value r_0 where $U_{\text{eff}}(r)$ has its minimum.
(B) Calculate $U_{\text{eff}}^{\text{min}} = U_{\text{eff}}(r_0)$.
(C) Assume an energy $E < 0$ and calculate the values of the turning points r_{min} and r_{max} .
(D) For which energy range do we have two real solutions?

Due in class, 5 points (2 for C, 1 for A, B, D).

- (20) Assume $0 < e < 1$ for the eccentricity and transform the elliptic equation

$$\frac{p}{r} = 1 + e \cos(\phi), \quad p > 0 \quad \text{into the form} \quad \frac{x'^2}{a^2} + \frac{y'^2}{b^2} = 1 .$$

This means, *derive* the definitions of x' , y' , major half-axis a and minor half-axis b in terms of x , y , p and e . Due Wednesday, October 19 before class, 8 points.

- (21) Use the inertial frame of §15 of the book and plot the orbits for the four initial conditions of the table in set 4 (best in one figure). Indicate the initial positions. Due October 24 before class, 12 points.

- (22) Calculate the turning point θ_{min} and θ_{max} for the spherical pendulum of the midterm for the special case $mgR = M_z^2/(4mR^2) = E/2$. Due in class (6 points).

Read Chapter V of Landau-Lifshitz up to §26, p.79.

- (23) Derive the results of problem 3 §21 of Landau and Lifshitz (frequency of oscillations). Due October 26 before class (4 points).

- (24) Calculate a and α of Eq. (22.4 and 1/2) of Landau and Lifshitz in terms of a and α of Eq.(22.4) of Landau and Lifshitz. Then, let $\gamma = \omega + \epsilon$. Taylor expand

$$x(t) = a \cos(\omega t + \alpha) + \frac{f [\cos(\gamma t + \beta) - \cos(\omega t + \beta)]}{m(\omega^2 - \gamma^2)}$$

to order ϵ and take the limit $\epsilon \rightarrow 0$. Due in class (4 points). Now October 28 before class.

- (25) Calculate the normal modes (eigenfrequencies) of the pendulum of problem 8, set 2. Due October 31 before class (4 points).