Intermediate Mechanics II — PHY 4936

Midterm Exam October 7, 2011

PROBLEM 1 (33 points)

The potential of the 1D harmonic oscillator is $U(x) = k\,x^2/2$.

1. Write down the Lagrangian.
2. Write down the Euler-Lagrange equation.
3. Solve the Euler-Lagrange equation. Express integration constants through time zero initial conditions x_0 and $\dot{x}_0$.

PROBLEM 2 (34 points)

Consider a point mass m on the surface of a sphere of radius R under the influence of gravity $-g\hat{z}$ (spherical pendulum).

1. Write down the Lagrange function using spherical coordinates.
2. Identify the conservation laws.
3. Find the special solutions for $\theta =$ constant. Describe this motion.

PROBLEM 3 (33 points)

Assume a Lagrangian $L = L(\{q_i\}, \{\dot{q}_i\}, t)$ where q_i, $i = 1, \ldots, n$ are generalized coordinates, $\dot{q}_i$, $i = 1, \ldots, n$ are generalized velocities and t is the time.

1. Write down the principle of least action.
2. Derive the Euler-Lagrange equations from the principle of least action.
3. Assume that the Lagrangian is invariant under translation $q_k \to q'_k = q_k + \epsilon_k$ of one or more generalized coordinates q_k. Find the corresponding conserved quantities.