FINAL – PHY 4936 (December 15, 2011)

PROBLEM 1 (25 points)

The Lagrangian of the 1D harmonic oscillator is

$$L = \frac{1}{2}\, m\, \dot{x}^2 - \frac{1}{2}\, k\, x^2 \ .$$

1. Use the definition of the generalized momentum to find the momentum p.
2. Write down the Hamiltonian $H(p,x)$.
3. Write down Hamilton's equations.
4. Show that Hamilton's equations give Newton's force law.

PROBLEM 2 (25 points)

Use the Lagrangian formalism to derive angular momentum conservation from the isotropy of space.

PROBLEM 3 (25 points)

Consider the effective potential of the Kepler problem

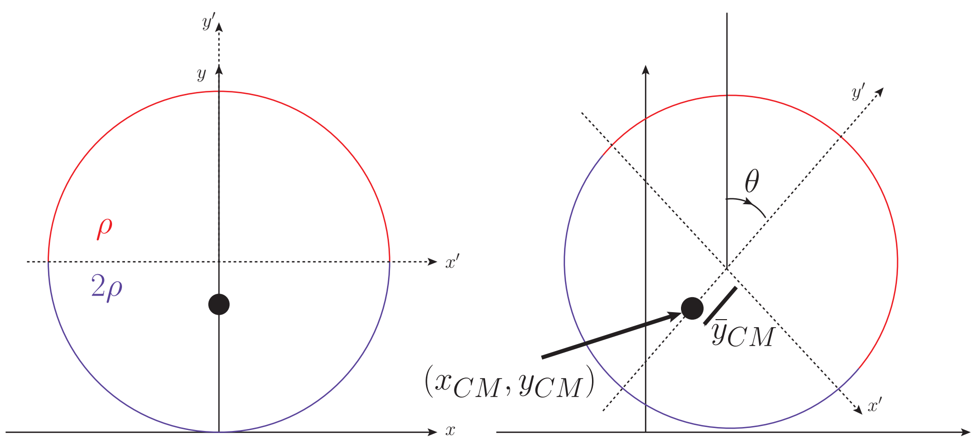
$$U_{\rm eff}(r) = -\frac{\alpha}{r} + \frac{L^2}{2\, m\, r^2} \ .$$

(A) Calculate the value r_0 where $U_{\rm eff}(r)$ has its minimum.
(B) Calculate $U^{\rm min}_{\rm eff} = U_{\rm eff}(r_0)$.
(C) Assume an energy $E < 0$ and calculate the turning points $r_{\rm min}$ and $r_{\rm max}$.
(D) For which energy range do we have two real solutions for $r_{\rm min}$ and $r_{\rm max}$?

PROBLEM 4 (25 points)

Consider a thin disk composed of two homogeneous halves connected along a diameter of the disk. As shown in the figure (see backside), one half has density ρ and the other has density 2ρ.

1. Find the center of mass (CM) in the body frame. Do these values change when the disk rolls?
2. Let R be the radius of the disk and θ be the angle defined in the figure. Find the CM position in the lab frame, x_{CM}, y_{CM}, as function of θ.
3. Use the parallel axis theorem to calculate the moment of inertia I^{CM}_3 for the axis perpendicular through the CM of the disk.
4. Write down the Lagrangian as function of the so far calculated quantities (assume gravity in the y direction of the lab frame). It is **not** not asked to evaluate the Lagrangian further (as it is done in a homework solution).



y'
y
ρ
x'
2ρ
x
y'
θ
$\bar{y}_{CM}$
(x_{CM}, y_{CM})
x'