HOME AND CLASS WORK – SET 1

Solution for assignment 5.

1. The Lagrangian is

$$L = \frac{1}{2}\,\dot{x}^2 - \frac{1}{2}\,x^2 \ .$$

Thus, the Euler-Lagrange equation becomes

$$0 = \frac{\partial L}{\partial x} - \frac{d}{dt}\,\frac{\partial L}{\partial \dot{x}} = -x - \ddot{x}$$

and the general solution is given by $x(t) = A\,\sin(t) + B\,\cos(t)$. The integration constants are determined by the boundary conditions

$$x(0) = B = 0 \ \text{ and } \ x(\pi/2) = A = 1$$

so that the exact path is

$$x(t) = \sin(t) \ .$$

For this path the action is given by

$$S\,[x(t)] = \frac{1}{2}\int_0^{\pi/2} dt\,\left[\cos^2(t) - \sin^2(t)\right] = \frac{1}{2}\int_0^{\pi/2} dt\,\cos(2t) = 0 \ .$$

2. For a linear path $x(t) = a + b\,t$ subject to the boundary conditions:

$$x(0) = a = 0 \ \text{ and } \ x(\pi/2) = b\,\frac{\pi}{2} = 1 \ \Rightarrow \ b = \frac{2}{\pi}$$

so that this path is

$$x(t) = \frac{2}{\pi}\,t \ .$$

For this path the action is given by

$$\begin{aligned} S\,[x(t)] &= \frac{1}{2}\int_0^{\pi/2} dt\,\left[\left(\frac{2}{\pi}\right)^2 - \left(\frac{2}{\pi}\right)^2\,t^2\right] = \frac{2}{\pi^2}\int_0^{\pi/2} dt\,\left(1 - t^2\right) \\ &= \frac{2}{\pi^2}\left[\frac{\pi}{2} - \frac{1}{3}\left(\frac{2}{\pi}\right)^3\right] = \frac{1}{\pi}\left(1 - \frac{\pi^2}{12}\right) = \frac{1}{\pi} - \frac{\pi}{12} = 0.0565\ldots \ . \end{aligned}$$

3. Assuming that the previous result is in $J \cdot s$, we find for the large number

$$S/\hbar \approx 5.37 \times 10^{32} \ .$$