Solution for assignment 14: Problems 1 and 2, Landau-Lifshitz p.24.

1. The Lagrangians are

$$\begin{aligned} L &= \frac{1}{2}\, m \left(\frac{d\vec{r}}{d\, t}\right)^2 - U\,, \\ L' &= \frac{1}{2}\, m' \left(\frac{d\vec{r}}{d\, t'}\right)^2 - U'\end{aligned}$$

After substituting $t' = t\,\sqrt{m'/m}$ in L' the Lagrangians agree and the same paths are obtained for

$$t'/t \;=\; \sqrt{m'/m}\;.$$

2. The Lagrangians are

$$\begin{aligned} L &= \frac{1}{2}\, m \left(\frac{d\vec{r}}{d\, t}\right)^2 - U\,, \\ L' &= \frac{1}{2}\, m \left(\frac{d\vec{r}}{d\, t'}\right)^2 - U''\end{aligned}$$

Assume $U' = \alpha U$. After substituting $t' = t/\sqrt{\alpha}$ we obtain

$$L' \;=\; \alpha\, L\,.$$

Therefore the same paths are obtained for

$$t'/t \;=\; \alpha^{-1/2} \;=\; \sqrt{U/U'}\,.$$