Solution for assignment 15:
The total Energy the is

$$
\begin{equation*}
E=\frac{1}{2} \dot{x}^{2}+\frac{1}{2} \frac{k}{x^{2}} . \tag{1}
\end{equation*}
$$

The initial values imply

$$
\begin{equation*}
E=\frac{1}{2} \frac{k}{x_{0}^{2}} \tag{2}
\end{equation*}
$$

Therefore,

$$
\begin{align*}
\dot{x}^{2} & =\frac{k}{m}\left(\frac{1}{x_{0}^{2}}-\frac{1}{x^{2}}\right)  \tag{3}\\
\dot{x} & = \pm \sqrt{\frac{k}{m x_{0}^{2}}} \frac{\sqrt{x^{2}-x_{0}^{2}}}{x} . \tag{4}
\end{align*}
$$

Separation of variables gives

$$
\begin{equation*}
\pm \sqrt{\frac{k}{m x_{0}^{2}}} \int_{0}^{t} d t^{\prime}= \pm \sqrt{\frac{k}{m x_{0}^{2}}} t=\int_{x_{0}}^{x} d x^{\prime} \frac{x^{\prime}}{\sqrt{x^{\prime 2}-x_{0}^{2}}}=\sqrt{x^{2}-x_{0}^{2}} . \tag{5}
\end{equation*}
$$

Therefore, the solution is

$$
\begin{equation*}
x(t)=\sqrt{\frac{k}{m x_{0}^{2}} t^{2}+x_{0}^{2}} . \tag{6}
\end{equation*}
$$

