ADVANCED DYNAMICS — PHY 4936

HOME AND CLASS WORK – SET 5

Solution for assignment 25:

Normal Modes of Double Pendulum (continuation of 8).

For small oscillation we have derived the Euler-Lagrange equations, which read in matrix notation

$$\begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} \ddot{\phi} \\ \ddot{\psi} \end{pmatrix} + \begin{pmatrix} 2g/l & 0 \\ 0 & g/l \end{pmatrix} \begin{pmatrix} \phi \\ \psi \end{pmatrix} \;=\; 0 \;.$$

This is solved by the exponential ansatz (physical is the real part of the solution):

$$\Phi(t) = \begin{pmatrix} \phi \\ \psi \end{pmatrix} \;=\; e^{i\,\omega\,t} \begin{pmatrix} \phi_0 \\ \psi_0 \end{pmatrix} \quad \Rightarrow \quad \begin{pmatrix} \ddot{\phi} \\ \ddot{\psi} \end{pmatrix} \;=\; -\omega^2\, e^{i\,\omega\,t} \begin{pmatrix} \phi_0 \\ \psi_0 \end{pmatrix}$$

$$\begin{pmatrix} -2\omega^2 + 2g/l & -\omega^2 \\ -\omega^2 & -\omega^2 + g/l \end{pmatrix} \begin{pmatrix} \phi \\ \psi \end{pmatrix} \;=\; 0 \;.$$

$$0 \;=\; \det \begin{vmatrix} -2\omega^2 + 2g/l & -\omega^2 \\ -\omega^2 & -\omega^2 + g/l \end{vmatrix} = \omega^4 - \frac{4g}{l}\,\omega^2 + \frac{2g^2}{l^2}$$

with eigenmodes

$$\omega_\pm \;=\; \sqrt{\frac{g}{l}}\,\sqrt{2 \pm \sqrt{2}} \;.$$