**Electrodynamics A (PHY 5346) Fall 2016 Solutions**

**Set 1:**

1. Non-relativistic distance measurement

With $x = vt$ the first event is conveniently chosen $x_0 = 0$ for $t = 0$.

In the non-relativistic limit $u' = u - v$ and the elastic bounce back velocity in the frame $K'$ is $-u'$. So it is in frame $K$

$$-u' + v = -u + 2v\,.$$

Hence, $u > 2v$ is required for the ball to come back at all. Assume the ball is emitted at time $t^e$. It will hit $O'_0$ at a time $t_1$ determined by

$$x_1 = v\,t_1 = u\,(t_1 - t^e) \;\Rightarrow\; t_1 = \frac{u\,t^e}{u - v} \;\text{ and }\; x_1 = v\,t_1 = \frac{u\,v\,t^e}{u - v}\,.$$

With $t_1^r$ defined as the time at which the ball is received back by $O_0$,

$$(u - 2v)\,(t_1^r - t_1) = x_1 = v\,t_1\,,$$

$$(u - 2v)\,t^r = (u - 2v)\,t_1 + v\,t_1 = (u - v)\,t_1 = u\,t^e\,.$$

Introducing the time difference $\triangle t = t^r - t^e$ between received and emitted,

$$u\,(t^r - t^e) = u\,\triangle t = 2v\,t^r = 2v\,(t^e + \triangle t)\,,$$

with the final result

$$v = \frac{u\,\triangle t}{2\,(t^e + \triangle t)} \;\to\; \frac{u}{2} \text{ for } \triangle t \to \infty\,. \tag{1}$$

This ought to be compared with the equation for a light signal, which is for the situation $x = v\,t$ (i.e., $x_0 = 0$) derived from

$$x_1 = v\,(t^e + \triangle t/2) = \frac{c\,\triangle t}{2} \;\Rightarrow\; v = \frac{c\,\triangle t}{2\,(t^e + \triangle t/2)} \;\to\; c \text{ for } \triangle t \to \infty\,.$$

Due to the asymmetry of the outward and return travel in the non-relativistic case, equation (1) is more difficult to derive than the one for light. Besides, there is a crucial difference by a factor of two for $\triangle t$ in the denominator. A return signal is only received for $u > 2v$, whereas $c > v$ is sufficient for light signals.

2. Galilei transformations

   The equation
$$c^2t^2 - \vec{x}^2 = 0 \quad \text{in } K$$
   is derived from
$$\vec{x} - \vec{c}\,t = 0$$
   which holds in $K$ for the propagation of the light in the direction $\vec{c}$. This equation transforms into
$$\vec{x}^{\,'} - \vec{v}t - \vec{c}\,t = 0 \quad \text{in } K' .$$
   Hence, in $K'$
$$\vec{x}^{\,'} - \vec{c}^{\,'}t = 0 \quad \text{with} \quad \vec{c}^{\,'} = \vec{c} + \vec{v} .$$
   In $K'$ the speed of light $c' = \sqrt{c'^2}$ is no longer a constant, but $c'^2 = (\vec{c}+\vec{v})^2$ depends on the angle between $\vec{c}$ and $\vec{v}$.

3. Contractions.

   $(1),\ (2)$ and $(3):\ \ 25-1-4-9=11;$

   $(4)$ and $(5):\ \ 25-9-16=0;$

   $(6):\ \ 25-9-16=-4;$

   $(7):\ \ 25-0-9-16=0.$

4. Time and relativistic distance measurements by light signals.

   A sufficiently accurate approximation of The elapsed time is given by
$$\frac{2,757,790}{9,193}\,[s] \approx 300\,[s] .$$
   (1) The relation $x = \triangle t/2$ gives for all three times
$$10^{-3} \times 3 \times 10^5\,[km] \ = \ 300\,[km] .$$
   Therefore, $O_2$ is at rest with respect to $O_1$.

   (2) We find the following positions at the following times:
$$\begin{aligned} x_1 &= 300\,[km] \quad \text{at} \quad t_1 = 1.001\,[s] , \\ x_2 &= 600\,[km] \quad \text{at} \quad t_2 = 2.002\,[s] , \\ x_3 &= 900\,[km] \quad \text{at} \quad t_3 = 3.003\,[s] . \end{aligned}$$

This gives the velocities

$$
\begin{aligned}
v_{21} &= \frac{x_2 - x_1}{t_2 - t_1} = \frac{300\,[km]}{(2.002 - 1.001\,[s]} = \frac{300\,[km]}{1.001\,[s]} \approx 299.7\,[km/s]\,, \\
v_{32} &= \frac{x_3 - x_2}{t_3 - t_2} = \frac{300\,[km]}{(3.003 - 2.002)\,[s]} = \frac{300\,[km]}{1.001\,[s]} \approx 299.7\,[km/s]\,.
\end{aligned}
$$

So, the results are consistent with the idea that $O_2$ is at rest in an inertial frame, which moves with about $299.7\,[km/s]$ with respect to the inertial frame of $O_1$. The position of $O_2$ with respect to $O_1$ is then given by

$$x(t) = x_0 + vt = \frac{300\,[km]}{1.001\,[s]}\,t\,,$$

where $x_0 = 0$ follows from $x(t_1) = 300\,[km]$.

(3) We find again the positions $x_1 = 300\,[km]$ at $t_1 = 1.001\,[s]$ and $x_2 = 600\,[km]$ at $t_2 = 2.002\,[s]$, which gives again $v_{21} = (300/1.001)\,[km/s]$, but now

$$x_3 = 1200\,[km] \quad \text{at} \quad t_3 = 3.004\,[s]\,,$$

which gives

$$v_{32} = \frac{x_3 - x_2}{t_3 - t_2} = \frac{600\,[km]}{1.002\,[s]} \approx 588.2\,[km/s]\,.$$

As $v_{21}$ and $v_{32}$ disagree, $O_2$ cannot be at rest in an inertial frame.

Note: If one wants to find suitable $t_2$, $t_1$, $\triangle t_2$ and $\triangle t_1$ values for a given speed $v$, this can be done using the formula

$$t_2 - t_1 = \frac{c}{2}\,\frac{\triangle t_2 - \triangle t_1}{v}\,.$$

There many solutions. For instance with, besides $v$, also $\triangle t_2 - \triangle t_1 > 0$ given, any $t_2 - t_1$ difference that matches will do. For instance, for $\triangle t_2 - \triangle t_1 = 2 \times 10^3 [s]$ and $t_1 = 1.001\,[s]$, $t_2 - t_1 = 1\,[s]$ (not $1.001\,[s]$) is needed to get precisely $300\,[km/s]$. Means starting time for $t_2$ at $1.999\,[s]$, receiving time at $2.003\,[s]$..