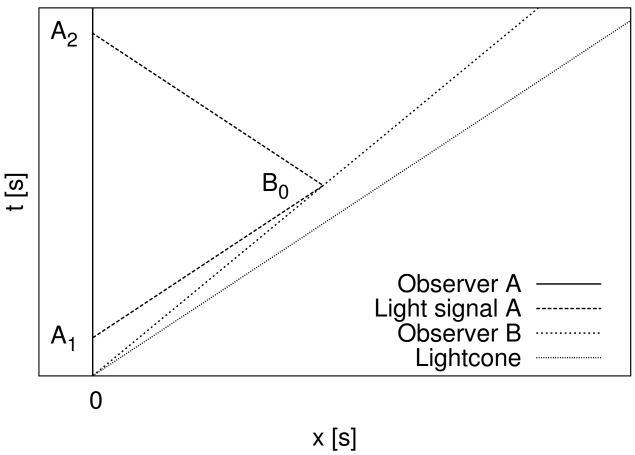
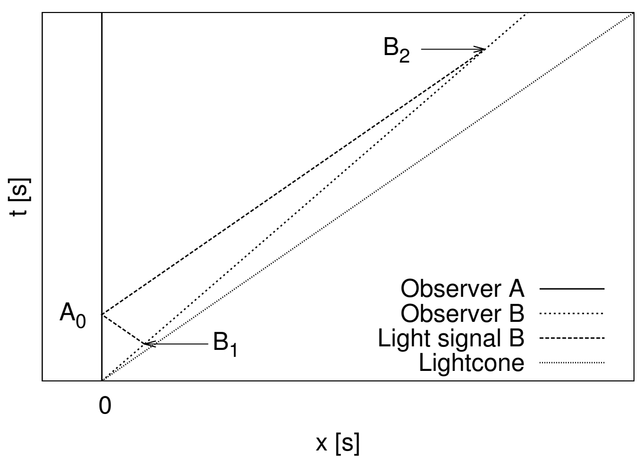


Minkowski space in which observer A is at rest and flashes a light signal at observer B, who moves with speed $4c/5$ and flashes the signal back.



Minkowski space in which observer A is at rest and observer B moves with speed $4c/5$. Observer B flashes a light signal at observer A, who flashes it back.

Electrodynamics A (PHY 5346) Fall 2016 Solutions

Set 2:

(5) Exercise: Time in Minkowski space.

We use natural units, $c = 1$, and give all results in units of seconds.

(a) It follows from $x^1 = 4\,t/5 = t - 15$ that the coordinates of B_0 are $(t, x) = (75, 60)$.
(b) The proper time of B is then $\tau = \sqrt{t^2 - x^2} = 45$.
(c) At position A_2 the time on the clock of A is $75 + 60 = 135$.
(d) The coordinates of position B_1 are $(t, x) = (t, 4t/5) = (25, 20)$ so that $\tau = \sqrt{t^2 - x^2} = 15$ holds.
(e) The light signal from B reaches A at time $25 + 20 = 45$ at position A_0, which agrees with the time of B at B_0.
(f) It follows from $x^1 = 4\,t/5 = t - 45$ that the coordinates of B_2 are $(t, x) = (225, 180)$ and the clock of B shows $\tau = \sqrt{t^2 - x^2} = 135$.
(g) The times agree. That has to be the case, because the travel of A in the rest frame of B does just mirror (opposite velocity signs) the travel of B in the rest frame of B.

(6) Exercise: Euclidean and hyperbolic rotations.

Transformation of the 2D Euclidean rotation.

$$\begin{pmatrix} x'^{\,1} \\ i\,x'^{\,0} \end{pmatrix} = \begin{pmatrix} \cosh(\zeta) & i\,\sinh(\zeta) \\ -i\,\sinh(\zeta) & \cosh(\zeta) \end{pmatrix} \begin{pmatrix} x^1 \\ i\,x^0 \end{pmatrix}$$

and in components

$$x'^{\,1} = \cosh(\zeta)\,x^1 - \sinh(\zeta)\,x^0\,,$$
$$i\,x'^{\,0} = -i\,\sinh(\zeta)\,x^1 + i\,\cosh(\zeta)\,x^0\,,$$

or, equivalently,

$$x'^{\,0} = +\cosh(\zeta)\,x^0 - \sinh(\zeta)\,x^1\,,$$
$$x'^{\,1} = -\sinh(\zeta)\,x^0 + \cosh(\zeta)\,x^1\,,$$

$$\begin{pmatrix} x'^{\,0} \\ x'^{\,1} \end{pmatrix} = \begin{pmatrix} \cosh(\zeta) & -\sinh(\zeta) \\ -\sinh(\zeta) & \cosh(\zeta) \end{pmatrix} \begin{pmatrix} x^0 \\ x^1 \end{pmatrix}$$