Electrodynamics A (PHY 5346) Fall 2016 Classwork and Homework

Every exercise counts 10 points unless stated differently.

Set 1:

(1) Exercise E.2: A non-relativistic distance measurement. Homework, due 9/13/2016 before class.
(2) Exercise E.4: Non-relativistic Galilei transformation. Homework, due 9/13/2016 before class.
(3) Exercise E.5: $x_\alpha x^\alpha$ contractions. Homework, due 9/13/2016 before class.
(4) Time and relativistic distance measurements. Classwork, due 9/1/2016 in class.
A Cesium clock counts 2,757,789,531,312 cycles (starting at 0). Find the elapsed time in seconds (you may round to the nearest integer number)?
An observer O_1 is located in an inertial system and flashes at times 1, 2, 3 ... $[s]$ light signals towards another observer O_2, who reflects them back with a mirror. Assume that the returned signals are received by O_1 at the following times:
(1) 1.002, 2.002, 3.002 ... $[s]$,
(2) 1.002, 2.004, 3.006 ... $[s]$,
(3) 1.002, 2.004, 3.008 ... $[s]$.
Determine for each case whether the data are consistent with assuming that O_2 is also in an inertial system. If this is the case, write down the equation for the distance of O_2 as function of the time as seen by O_1. Approximate the speed of light by $300,000\,[km/s]$, but perform all calculations with a precision of at least four digits.

Set 2:

(5) Exercise E.6: Time in Minkowski space. Homework, due 9/15/2016 before class.
(6) Exercise E.8: Euclidean and hyperbolic rotations. Homework, due 9/15/2016 before class.

Set 3:

(7) Exercise E.10 without part (5): Lorentz group Lie matrix generator. Classwork, 5 points, due 9/15/2016 in class.
(8) Exercise E.10 part (5). Homework, 5 points, due 9/22/2016 before class.
(9) Exercise E.11: A rotation and a boost generator. Homework, due 9/22/2016 before class.
(10) Exercise E.12: Four-dimensional Levi-Civita tensor. Homework, due 9/22/2016 before class.
(11) Exercise E.14: Addition theorem for transverse velocity components. Homework, due 9/22/2016 before class.

Set 4:

(12) Exercise E.17: End of spacetrip. Homework, due 9/29/2016 before class.
(13) Exercise E.19: Redshift. Homework, due 9/29/2016 before class.
(14) Exercise E.21: Relativistic energy-momentum conservation. Homework, due 9/29/2016 before class.
(15) Electromagnetic field tensor, due 9/27/2016 in class.
(A) Compare $\partial_\alpha F^{\alpha\beta} = (4\pi/c)\, J^\beta$ for $\beta = 0$ with the inhomogeneous Maxwell equations

$$\nabla \cdot \vec{E} = 4\pi\rho = \frac{4\pi}{c}\, J^0, \quad \nabla \times \vec{B} - \frac{1}{c}\frac{\partial \vec{E}}{\partial t} = \frac{4\pi}{c}\, \vec{J}$$

to identify F^{00}, F^{10}, F^{20} and F^{30} in terms of $\vec{E}$ and $\vec{B}$. Continue with $\beta = 1$, then $\beta = 2$. Do you need $\beta = 3$ too?
(B) The result of (A) translates into (see the lecture notes)

$$({}^*F^{\alpha\beta}) = \begin{pmatrix} 0 & {}^*F^{01} & {}^*F^{02} & {}^*F^{03} \\ {}^*F^{10} & 0 & {}^*F^{12} & {}^*F^{13} \\ {}^*F^{20} & {}^*F^{21} & 0 & {}^*F^{23} \\ {}^*F^{30} & {}^*F^{31} & {}^*F^{32} & 0 \end{pmatrix} = \begin{pmatrix} 0 & -B^x & -B^y & -B^z \\ B^x & 0 & E^z & -E^y \\ B^y & -E^z & 0 & E^x \\ B^z & E^y & -E^x & 0 \end{pmatrix} .$$

Show that this form and $\partial_\alpha\, {}^*F^{\alpha\beta} = 0$ imply the homogeneous Maxwell equations

$$\nabla \cdot \vec{B} = 0\,, \quad \nabla \times \vec{E} + \frac{1}{c}\frac{\partial \vec{B}}{\partial t} = 0\,.$$

Fix β to 0, 1, 2, 3 and discuss each case separately.

Set 5:

(16) Exercise E.18: Time dilation for a satellite. Homework, due 10/6/2016 before class.

(17) Exercise E.26: Homogeneous Maxwell equations. Homework, due 10/6/2016 before class.

(18) Exercise E.27: Lorentz transformations of electric and magnetic fields Homework, due 10/6/2016 before class.

Set 6:

(19) Exercise E.31: Fields of a moving charge. Homework, due 10/13/2016 before class.

(20) Exercise: Divergence and Laplace operator. Classwork, due 10/13/2016 in class.

(a) Calculate $\nabla \cdot \vec{r}$.

(b) Calculate $\nabla \cdot \vec{r}\, f(r)$ as scalar function of r.

(c) Example: Calculate (b) for $f(r) = r^{n-1}$.

(d) Calculate the divergence of the electric field $\vec{E} = q\,\hat{r}/r^2$ for $r \neq 0$.

(e) Calculate $\nabla^2\,(q/r)$ for $r \neq 0$.

Set 7:

(21) Exercise E.36: Electric and magnetic fields for an infinitely long wire. Homework, due 10/20/2016 before class or till 10/21/2016 5pm in David Clarke's mailbox.

(22) Exercise E.37: Charge density from a potential. Homework, due 10/20/2016 before class or till 10/21/2016 5pm in David Clarke's mailbox.

(23) Exercise E.38: Symmetry of Dirichlet Green functions. Homework, due 10/20/2016 before class or till 10/21/2016 5pm in David Clarke's mailbox.

Set 8:

(24) Exercise E.40: Method of images for a plane. Homework, due 10/27/2016 before class.
(25) Exercise E.42: Potential from distinct BCs and half-spheres. Homework, due 10/27/2016 before class.
(26) Exercise E.46: Potential in a rectangular box. Homework, due 10/27/2016 before class.

Set 9:

(27) Exercise E.41: Potential over a plane. Due 11/3/2016 before class.
(28) Exercise E.44: Angled plates. Homework, due 11/3/2016 before class.
(29) Exercise: Expansion in cylindrical coordinates. Due 11/3/2016 before class:

For simplicity we use dimensionless numbers in the following problem. Consider a cylinder length $L = 1$ and radius $\rho_0 = 1$. The potential on the mantel and bottom surface is zero and on the top surface it is given by

$$R(\rho) \ = \ J_0\left(x_{01}\,\frac{\rho}{\rho_0}\right)\,,$$

where x_{01} is the first zero of the Bessel function $J_0(x)$.

(a) Find the potential everywhere inside the cylinder.
(b) Evaluate the potential numerically for $\rho = 0$, $z = 1/2$.
(c) Evaluate the potential numerically for $\rho = 1/2$, $z = 1/2$.

(30) Exercise: Classwork, due 11/3/2016 in class (5 points).

Use

$$L_- \ = \ \hat{x}\cdot\vec{L} - i\,\hat{y}\cdot\vec{L}$$

to find in spherical coordinates:

$$L_- \ = \ e^{-i\,\phi}\left(-\frac{\partial}{\partial\theta} + i\,\cot\theta\,\frac{\partial}{\partial\phi}\right)\,.$$

Set 10:

(31) Exercise E.39: Paul Trap. Due 11/10/2016 before class.

(32) Exercise: Potential of two point charges. Due 11/10/2016 before class: Two point charges are located on the z-axis: q_1 at z_1, $0 < z_1 < R$ and q_2 at $z_2 > R$. The resulting potential is

$$\Phi(\vec{x}\,) \;=\; \frac{q_1}{|\vec{x} - z_1\hat{z}|} + \frac{q_2}{|\vec{x} - z_2\hat{z}|}\,.$$

(a) Calculate q_2 and z_2 as functions of q_1, z_1 and R from the requirement that the potential is zero at $z = R$ and $z = -R$.

(b) Calculate the resulting potential on the surface of the sphere of radius R with center at $\vec{x} = 0$.

(33) Exercise E.49: Potential inside a hollow sphere with Dirichlet BC. Due 11/10/2016 before class.

Set 11:

(34) Exercise E.48: A symmetry relation for spherical harmonics. Due 11/17/2016 before class.

(35) Exercise E.51: Dirichlet Green function of a cylinder. Due 11/17/2016 before class.

(36) Exercise E.52: Point charge at the center of a rectangular box. Due 11/17/2016 before class.

Set 12:

(37) Exercise: Classwork, due 11/15/2016 in class.

(a) 10 points: Use Taylor series expansion to derive the Cartesian quadrupole expansion

$$\Phi(\vec{x}\,) \;=\; \int d^3x'\,\frac{\rho(\vec{x}\,')}{|\vec{x} - \vec{x}\,'\,|} \;=\; \frac{q}{r} + \frac{\vec{p}\cdot\vec{x}}{r^3} + \frac{1}{2}\sum_{i=1}^{3}\sum_{j=1}^{3} Q^{ij}\,\frac{x^i\,x^j}{r^5} + \dots\,.$$

(b) Due 11/29/2016, 10 points: Derive the same result by expressing the spherical multipoles q_{20}, q_{21} and q_{22} in terms of Cartesian multipoles Q^{ij}. Compare (E.56) of the lecture notes.

(38) Exercise E.55: Number of Cartesian multipoles. Due 11/29/2016 before class.

(39) Exercise E.57: Quadrupole moment of an ellipsoid. Due 11/29/2016 before class.

(40) Exercise E.58: Dielectric sphere. Due 11/29/2016 before class.

(41) Exercise E.34 (c): Classwork, 5 points.
Due 11/22/2016 in class.

Use

$$\vec{a} \times \vec{b} \; = \; \sum_{i=1}^{3} \sum_{j=1}^{3} \sum_{k=1}^{3} \epsilon^{ijk} \, \hat{x}^i \, \partial_j \, b^k$$

and

$$\left(\vec{a} \times \vec{b} \right)^k \; = \; \sum_{l=1}^{3} \sum_{m=1}^{3} \epsilon^{klm} \, \partial_l \, b^m$$

to derive

$$\nabla \times \left(\nabla \times \vec{b} \right) \; = \; \nabla \, \left(\nabla \cdot \vec{b} \right) - \nabla^2 \, \vec{b} \, .$$