**Electrodynamics A (PHY 5346) Fall 2016 Solutions**
**Set 13:**

(41) **Magnetic field in a hole of a conductor.**

We choose the axis of the conductor (and the hole) in the $\hat{z}$ direction. The current density is $\vec{J} = J\,\hat{z}$ with

$$J \;=\; \frac{I}{A} \;=\; \frac{I}{\pi\,(R^2 - r^2)}\;.$$

The trick is to write the magnetic field as a linear combination $\vec{B} = \vec{B}_1 + \vec{B}_2$ where $\vec{B}_1$ correspond to the current density $\vec{J}_1 = \vec{J}$ flowing over the *entire* cross section of the conductor, and $\vec{B}_2$ to the current density $\vec{J}_2 = -\vec{J}$ flowing over the cross section of the cylindrical hole. Due to the linearity of Ampére's law this give the same magnetic field as the original arrangement. With $i = 1$, 2 and $\vec{J}_i = J_i\,\hat{z}$ we have

$$2\pi\,\rho_i\,B_i \;=\; \oint_C \vec{B}_i \cdot d\vec{l} \;=\; \frac{4\pi}{c}\int_S J_i\,da \;=\; \frac{4\pi^2}{c}\,\rho_i^2\,J_i\,,$$

where the integration contours are circles of radii $\rho_i$ about the axis of the conductor ($i = 1$) and the cylindrical hole ($i = 2$). The magnetic field vectors are

$$\vec{B}_i \;=\; \frac{2\pi}{c}\,\rho_i\,J_i\,\hat{\phi}_i\,, \qquad \hat{\phi}_i \;=\; -\sin(\phi_i)\,\hat{x} + \cos(\phi_i)\,\hat{y}$$

where have choosen the center of the hole on the $x$ axis and defined the angles $\phi_i$ by $\cos(\phi_i) = \vec{\rho}_i \cdot \hat{x}$, where $\vec{\rho}_i$ are the vectors from the axis of the conductor ($i = 1$) and from the axis of the hole ($i = 2$) to the (same) point at which we calculate the magnetic field. The magnetic field becomes

$$\vec{B} \;=\; \frac{2\pi}{c}\,J\,\{\rho_1\,[-\sin(\phi_1)\,\hat{x} + \cos(\phi_1)\,\hat{y}\,] - \rho_2\,[-\sin(\phi_2)\,\hat{x} + \cos(\phi_2)\,\hat{y}\,]\}$$

From the geometry we have for the components of the position vector

$$x = \rho_1\,\cos(\phi_1) = d + \rho_2\,\cos(\phi_2) \;\text{ and }\; y = \rho_1\,\sin(\phi_1) = \rho_2\,\sin(\phi_2)\;.$$

Therefore,

$$\vec{B} \;=\; \frac{2\pi}{c}\,J\,d\,\hat{y} \;=\; \frac{2\,I\,d}{c\,(R^2 - r^2)}\,\hat{y}$$

holds. In the cylindrical hole there is a constant field in the $\hat{y}$ direction.