Mathematical Physics — PHZ 3113

Levi-Cevita Tensor Homework 1
(January 18, 2013)

1. Use the $3D$ identity

$$\sum_{i=1}^{3} \epsilon_{ijk}\epsilon_{ilm} = \delta_{jl}\delta_{km} - \delta_{jm}\delta_{kl} \quad (1)$$

to calculate

$$(\hat{a} \times \hat{b}) \cdot (\hat{a} \times \hat{b}) \quad\quad (2)$$

where $\hat{a}$ and $\hat{b}$ are unit vectors. Eliminate all $\hat{a} \cdot \hat{b}$ in favor of $\cos\theta$. Use the notation

$$\hat{a} = \begin{pmatrix} \tilde{a}_1 \\ \tilde{a}_2 \\ \tilde{a}_3 \end{pmatrix} \quad \text{and} \quad \hat{b} = \begin{pmatrix} \tilde{b}_1 \\ \tilde{b}_2 \\ \tilde{b}_3 \end{pmatrix}. \quad (3)$$

2. Use the previous result to show $\vec{a} \times \vec{b} = a\,b\,\sin\theta$ (book p.21).