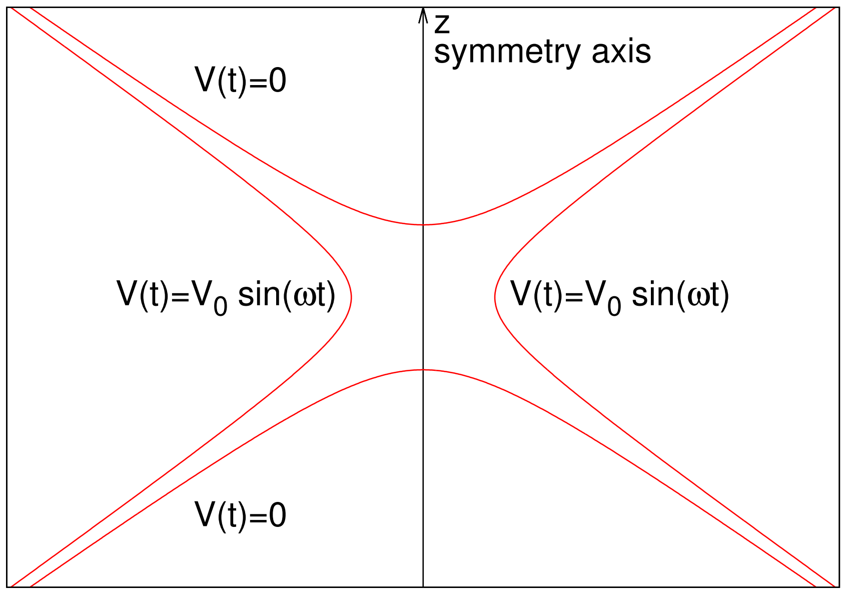

Dirichlet Boundary Condition: Paul Trap

A Paul Trap (Physics Nobel Prize 1989) is used to confine charged particles. Let us consider a cylindrical symmetric trap, which is conveniently described in cylindrical coordinates (ρ, θ, z). The trap electrodes are hyperbolas of revolution about the z axis. The figure depicts a cross section of the trap.

By rotation about the z axis the two grounded hyperbolas with potential $V(t) = 0$ form the upper and lower end electrodes, defined by the equation $z^2 = \rho^2/2 + d^2$.

The hyperbolas with potential $V(t) = V_0 \sin(wt)$ get connected and form the ring electrode, defined by the equation $\rho^2/2 = z^2 + d^2/2$.

Suppose $\omega \ll c/d$ (c is the speed of light). Use the quasi-static approximation and Poisson's equation

$$\nabla^2 \Phi = 0$$

to calculate the electric field inside the trap.

Hint: Find first the most general potential Φ that fulfills the boundary condition on the end electrodes. Proceed to implement the boundary condition of the ring electrode and show that the obtained expression is a solution of the Poisson equation.