Mathematical Physics — PHZ 3113

**Divergence,** (January 25, 2013)

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1. Solution

$$\nabla \cdot \vec{r} \;=\; \sum_{i=1}^{3} \frac{\partial x_i}{\partial x_i} \;=\; 3\,. \qquad (1)$$

2. Solution

$$\nabla \cdot \vec{r}\, f(r) \;=\; f(r)\, \nabla \cdot \vec{r} + \vec{r} \cdot \nabla f$$

$$=\; 3\, f(r) + \vec{r} \cdot \hat{r}\, \frac{df}{r} \;=\; 3\, f(r) + r\, \frac{df}{r}\,. \quad (2)$$

3. Example: Calculate (2) for $f(r) = r^{n-1}$.
   Solution:

$$\nabla \cdot \vec{r}\, r^{n-1} \;=\; 3\, r^{n-1} + r\, \frac{dr^{n-1}}{d\, r}$$

$$=\; 3\, r^{n-1} + (n-1)\, r^{n-1} \;=\; (n+2)\, r^{n-1}\,. \quad (3)$$

4. Solution for $\vec{r} \neq \vec{0}$

$$\nabla \cdot \vec{E} \;=\; \nabla \cdot \frac{q\, \vec{r}}{4\pi\, \epsilon_0\, r^3} \qquad (4)$$

$$=\; (-2+2)\, \frac{q}{4\pi\, \epsilon_0\, r^3} \;=\; 0\,.$$