Mathematical Physics — PHZ 3113

Gradient, (January 23, 2013)

Group #

Participating students (print):

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1. Calculate

$$\frac{\partial}{\partial x_i} x_j \, . \tag{1}$$

It holds

$$\frac{\partial}{\partial x_i} x_j \; = \; \delta_{ij} \, . \tag{2}$$

2. Calculate

$$\frac{\partial}{\partial x_i} \sum_{j=1}^{n} x_j^2 \, . \tag{3}$$

Solution:

$$\frac{\partial}{\partial x_i} \sum_{j=1}^{n} x_j^2 \; = \; \sum_{j=1}^{n} \frac{dx_j^2}{d\,x_j} \frac{\partial x_j}{\partial x_i} =$$
$$\sum_{j=1}^{n} 2\, x_j \, \delta_{ij} \; = \; 2\, x_i \, . \tag{4}$$

3. Calculate

$$\frac{\partial}{\partial x_i} \sqrt{\sum_{j=1}^{n} x_j^2}\,. \tag{5}$$

Solution:

$$\frac{\partial}{\partial x_i} \sqrt{\sum_{j=1}^{n} x_j^2} = \frac{\partial \sqrt{\sum_{j=1}^{n} x_j^2}}{\partial \sum_{j=1}^{n} x_j^2} \frac{\partial}{\partial x_i} \sum_{j=1}^{n} x_j^2$$
$$= \frac{1}{2\sqrt{\sum_{j=1}^{n} x_j^2}}\, 2\, x_i = \frac{x_i}{\sqrt{\sum_{j=1}^{n} x_j^2}}\,. \tag{6}$$

4. Calculate

$$\nabla\, r\,. \tag{7}$$

Solution:

$$\nabla\, r = \sum_{i=1}^{n} \hat{x}_i \frac{\partial}{\partial x_i} \sqrt{\sum_{j=1}^{n} x_j^2} \tag{8}$$
$$= \sum_{i=1}^{n} \hat{x}_i \frac{x_i}{r}, = \frac{\vec{r}}{r} = \hat{r}\,.$$

5. Calculate

$$\nabla\, f(r)\,. \tag{9}$$

Solution:

$$\nabla\, f(r) = \frac{df}{dr}\, \nabla\, r = \hat{r}\, \frac{df}{dr}\,. \tag{10}$$

6. Example: Calculate the electric field $\vec{E}$ from a given central potential:

$$\vec{E} = -\nabla\,\Phi(r)\,, \quad \Phi(r) = \frac{q}{4\pi\,\epsilon_0\,r}\,. \tag{11}$$

Solution:

$$\vec{E} = \frac{q\,\hat{r}}{4\pi\,\epsilon_0\,r^2}\,. \tag{12}$$

7. Calculate

$$\nabla\,\left(\vec{n}\cdot\vec{r}\right) \tag{13}$$

where $\vec{n}$ is a constant vector.

Solution:

$$\nabla\,\left(\vec{n}\cdot\vec{r}\right) \;=\; \sum_{i=1}^{n} \hat{x}_i\,\frac{\partial}{\partial x_i}\,\sum_{j=1}^{n} n_j\,x_j \;=\; \sum_{i=1}^{n}\sum_{j=1}^{n} \hat{x}_i\,n_j\,\delta_{ij} \;=\; \sum_{i=1}^{n} \hat{x}_i\,n_i \;=\; \vec{n}\,. \tag{14}$$

Describe the solutions $\vec{r}$ for $\vec{n}\cdot\vec{r} = 0$ in words.

The $\vec{r}$ vectors span the surface, which is perpendicular to $\vec{n}$ and goes through the origin $\vec{r} = 0$.