Mathematical Physics — PHZ 3113

Levi-Cevita Tensor Homework 2

(January 25, 2013)

1. Use the identity

$$\sum_{i=1}^{3} \epsilon_{ijk}\epsilon_{ilm} = \delta_{jl}\delta_{km} - \delta_{jm}\delta_{kl} \qquad (1)$$

to eliminate the vector products from the expression

$$\vec{a} \times \left(\vec{b} \times \vec{c}\right) \qquad (2)$$

Solution (compare book p.33):

$$\begin{aligned}
& \vec{a} \times \left(\vec{b} \times \vec{c}\right) \\
&= \sum_i \sum_j \sum_k \sum_l \sum_m \epsilon_{ijk} \hat{x}_i a_j \epsilon_{klm} b_l c_m \\
&= \sum_i \sum_j \sum_k \sum_l \sum_m \epsilon_{kij} \epsilon_{klm} \hat{x}_i a_j b_l c_m \\
&= \sum_i \sum_j \sum_l \sum_m \left(\delta_{il}\delta_{jm} - \delta_{im}\delta_{jl}\right) \hat{x}_i a_j b_l c_m \\
&= \vec{b}\,(\vec{a} \cdot \vec{c}) - \vec{c}\left(\vec{a} \cdot \vec{b}\right) .
\end{aligned}$$

2. Use the definition

$$\left(\vec{b} \times \vec{c}\right)_i = \sum_j \sum_k \epsilon_{ijk} b_j c_k \qquad (3)$$

of the i^{th} component of the vector product $\vec{b} \times \vec{c}$ to proof

$$\vec{a} \cdot \left(\vec{b} \times \vec{c}\right) = \vec{b} \cdot (\vec{c} \times \vec{a}) = \vec{c} \cdot \left(\vec{a} \times \vec{b}\right) \ . \quad (4)$$

Solution (compare book p.30):

$$\vec{a} \cdot \left(\vec{b} \times \vec{c}\right) = \sum_i \sum_j \sum_k \epsilon_{ijk} a_i b_j c_k$$

$$= \sum_j \sum_k \sum_i \epsilon_{jki} b_j c_k a_i = \vec{b} \cdot (\vec{c} \times \vec{a})$$

$$= \sum_k \sum_i \sum_j \epsilon_{kij} c_k a_i b_j = \vec{c} \cdot \left(\vec{a} \times \vec{b}\right) \ .$$