Classwork 14

Calculate eigenvalues and eigenvectors of the matrix

$$A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}, \quad A\hat{V}_i = \lambda_i \hat{V}_i$$

Eigenvalues

Eigenvectors

Normalization:

$$\hat{V}_i \cdot \hat{V}_i = 1$$

Solution:

$$\det \begin{vmatrix} 1-\lambda & 0 & 1 \\ 0 & 1-\lambda & 0 \\ 1 & 0 & 1-\lambda \end{vmatrix} = (1-\lambda)^3 - 1 + \lambda \overset{!}{=} 0$$

$$= -\lambda^3 + 3\lambda^2 - 3\lambda + 1 - 1 + \lambda$$

$$\Rightarrow \lambda^3 - 3\lambda^2 + 2\lambda = 0 \quad \Rightarrow \underline{\underline{\lambda_1 = 0}}$$

Left: $\lambda^2 - 3\lambda + 2 = 0$

$$\lambda_{2/3} = +\frac{3}{2} \mp \sqrt{\left(\frac{3}{2}\right)^2 - 2} = \frac{3}{2} \mp \frac{1}{2}$$

$$\underline{\underline{\lambda_2 = 1,}} \quad \underline{\underline{\lambda_3 = 2}} \, .$$

Eigenvector $\hat{v}_1$: $\quad \hat{v}_1 = \begin{pmatrix} v_1^1 \\ v_1^2 \\ v_1^3 \end{pmatrix}$

From $A_1 \hat{v}_1 = 0$,

$$A_1 = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} \Rightarrow \begin{aligned} v_1^1 + v_1^3 &= 0 \\ v_1^2 &= 0 \\ v_1^1 + v_1^3 &= 0 \end{aligned}$$

Normalized solution: $\hat{v}_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$

Eigenvector $\hat{v}_2$: $A_2 \hat{v}_2 = 0$,

$$A_2 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \Rightarrow \begin{aligned} v_2^3 &= 0 \\ v_2^2 \cdot 0 &= 0 \\ v_2^1 &= 0 \end{aligned}$$

$$\hat{v}_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

Eigenvector $\hat{v}_3$: $A_3 \hat{v}_3 = 0$,

$$A_3 = \begin{pmatrix} -1 & 0 & 1 \\ 0 & -1 & 0 \\ 1 & 0 & -1 \end{pmatrix} \Rightarrow \begin{aligned} -v_3^1 + v_3^3 &= 0 \\ v_3^2 &= 0 \\ v_3^1 - v_3^3 &= 0 \end{aligned}$$

$$\hat{v}_3 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$