Classwork 10:

1. Use spherical coordinates to calculate the volume of a sphere of radius R.

$$V = \int_{x^2+y^2+z^2 \le R^2} dx\,dy\,dz = \int_0^R r^2 dr \int_0^\pi \sin\theta\, d\theta \int_0^{2\pi} d\phi$$

$$= 2\pi \, \frac{1}{3} r^3 \Big|_0^R \int_1^{-1} (-d\cos\theta) = \frac{4\pi}{3} R^3$$

Classwork 10: Matrices of dot Products

We had already

$$\begin{array}{c|cc} & \hat{x} & \hat{y} \\ \hline \hat{s} & \cos\phi & \sin\phi \\ \hat{\phi} & -\sin\phi & \cos\phi \end{array}$$

Now, fill out:

$$\begin{array}{c|cc} & \hat{s} & \hat{z} \\ \hline \hat{r} & \sin\theta & \cos\theta \\ \hat{\theta} & \cos\theta & -\sin\theta \end{array}$$

$$\begin{array}{c|ccc} & \hat{x} & \hat{y} & \hat{z} \\ \hline \hat{r} & \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ \hat{\theta} & \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ \hat{\phi} & -\sin\phi & \cos\phi & 0 \end{array}$$

Classwork 10

3. Use cylindrical coordinates in the plane to calculate

$$\int_{-\infty}^{+\infty} e^{-x^2}\, dx = \sqrt{\pi}$$

Proof:

$$\left(\int_{-\infty}^{+\infty} e^{-x^2} dx\right)^2 = \int_{-\infty}^{+\infty} dx\, e^{-x^2} \int_{-\infty}^{+\infty} dy\, e^{-y^2}$$

$$= \int_0^{\infty} r\,dr\, e^{-r^2} \int_0^{2\pi} d\phi = 2\pi \int_0^{\infty} \left(-\frac{1}{2}\right) \frac{d}{dr} e^{-r^2} dr$$

$$= -\pi e^{-r^2} \Big|_0^{\infty} = \underline{\underline{\pi}}$$