Mathematical Physics — PHZ 3113

Sailboat (Homework January 16, 2013)

One of the major inventions of mankind (not known in the antique) was the keel (or skeg in dinghies), which allows sailing boats to zigzag against the wind. Below you find a sketch for the basic idea, where

$$0 < \beta < \alpha < \frac{\pi}{2} \tag{1}$$

holds for the angles. Calculate the forward component $\vec{F}^b$ from the wind force $\vec{F}$ on the boat under the assumption that friction and perpendicular drift of the boat can be neglected.

Solution:

Let $\vec{F} = -F\,\hat{y}$, $F > 0$. From the figure we find for the unit vectors parallel and perpendicular to the sail

$$\hat{s}_{||} = \sin(\beta)\,\hat{x} - \cos(\beta)\,\hat{y}\,, \tag{2}$$

$$\hat{s}_{\perp} = \cos(\beta)\,\hat{x} - \sin(\beta)\,\hat{y}\,. \tag{3}$$

The projections of the force on these directions are

$$\vec{F}^s_{||} = \left(\hat{s}_{||}\cdot\vec{F}\right)\,\hat{s}_{||}\,, \tag{4}$$

$$\vec{F}^s_{\perp} = \left(\hat{s}_{\perp}\cdot\vec{F}\right)\,\hat{s}_{\perp} = \sin(\beta)\,F\,\hat{s}_{\perp}\,, \tag{5}$$

of which the $\vec{F}^s_{||}$ component can be discarded, because we neglect friction. The unit vector $\hat{s}_{\perp}$ can be expanded into the unit vector parallel and perpendicular to the boat:

$$\hat{s}_{\perp} = \sin(\alpha-\beta)\,\hat{b}_{||} - \cos(\alpha-\beta)\,\hat{b}_{\perp}\,. \tag{6}$$

As the drift of the boat is neglected, we are only interested in the component parallel to the boat, which is our result

$$\vec{F}^b_{||} = \sin(\alpha-\beta)\,\sin(\beta)\,F\,\hat{b}_{||}\,. \tag{7}$$