Mathematical Physics — PHZ 3113

Vectors 1 (Classwork January 7, 2013)

Group #

Participating students (print):

In the following $i = 1, \ldots, n$, $j = 1, \ldots, n$.

1. Let $\hat{x}_i$ and $\hat{x}_j$ be Cartesian unit vectors. It holds the relation

$$\hat{x}_i \cdot \hat{x}_j \;=\; \delta_{ij}\,. \tag{1}$$

Name the r.h.s. quantity: Kronecker delta.

2. Write down n-dimensional (nD henceforth) column vectors.

$$\vec{a} \;=\; \begin{pmatrix} a_1 \\ . \\ . \\ . \\ a_n \end{pmatrix}, \qquad \vec{b} \;=\; \begin{pmatrix} b_1 \\ . \\ . \\ . \\ b_n \end{pmatrix}. \tag{2}$$

3. Write down the scalar product.

$$\vec{a}\cdot\vec{b} = \sum_{i=1}^{n} a_i\, b_i\,. \tag{3}$$

4. Write down the scalar products

$$\vec{a}\cdot\vec{a} = \sum_{i=1}^{n} a_i\, a_i\,, \quad \vec{b}\cdot\vec{b} = \sum_{i=1}^{n} b_i\, b_i\,. \tag{4}$$

5. Give the definition of the magnitude of $\vec{a}$.

$$a = |\vec{a}| = \sqrt{\vec{a}\cdot\vec{a}}\,. \tag{5}$$

6. Express the unit vectors $\hat{a}$ and $\hat{b}$ through previously defined quantities.

$$\hat{a} = \frac{\vec{a}}{|\vec{a}|} = \frac{\vec{a}}{a}\,, \qquad \hat{b} = \frac{\vec{b}}{|\vec{b}|} = \frac{\vec{b}}{b}\,. \tag{6}$$

7. Use $\vec{b}$ to find a unit vector that is perpendicular to $\hat{a}$. Which condition has $\vec{b}$ to fulfill, so that this is possible?

$$\vec{a}_\perp \;=\; \vec{b} - \left(\vec{b}\cdot\hat{a}\right)\hat{a} \tag{7}$$

if the condition $\vec{b} \neq \left(\vec{b}\cdot\hat{a}\right)\hat{a}$ holds. Show $\vec{a}_\perp \cdot \vec{a} = 0$. It follows from

$$0 = \vec{a}_\perp \cdot \hat{a} = \left(\vec{b}\cdot\hat{a}\right) - \left(\vec{b}\cdot\hat{a}\right)\left(\hat{a}\cdot\hat{a}\right) . \tag{8}$$

8. Expand the vectors $\vec{a}$ and $\vec{b}$ in terms of the unit vectors $\hat{x}_i$.

$$\vec{a} \;=\; \sum_{i=1}^{n} a_i\,\hat{x}_i\,, \qquad \vec{b} \;=\; \sum_{j=1}^{n} b_j\,\hat{x}_j\,. \tag{9}$$

9. Calculate the scalar products for the r.h. sides of the previous equation and show that the results agrees with Eq. (3).

$$\begin{aligned}\vec{a}\cdot\vec{b} &= \sum_{i=1}^{n} a_i\,\hat{x}_i \cdot \sum_{j=1}^{n} b_j\,\hat{x}_j \\ &= \sum_{i=1}^{n}\sum_{j=1}^{n} a_i\,b_j\;\hat{x}_i\cdot\hat{x}_j \\ &= \sum_{i=1}^{n}\sum_{j=1}^{n} a_i\,b_j\,\delta_{ij} \;=\; \sum_{i=1}^{n} a_i\,b_i\,.\end{aligned}$$