Mathematical Physics — PHZ 3113
Midterm 1 (February 18, 2013)

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1. Calculate the gradient of the 3D potential (in arbitrary units) $\frac{1}{2} r^2$.

2. A force in 3D is (in arbitrary units) given by $\vec{F} = -r\,\vec{r}$. Use (with Einstein convention)

$$\nabla \times r\,\vec{r} \;=\; \epsilon_{ijk}\,\hat{x}_i\,\partial_j\,r\,x_k$$

to calculate the curl of this force.

3. Calculate the curl of the force

$$\vec{F} = x_2\,\hat{x}_1 + x_3\,\hat{x}_2 + x_1\,\hat{x}_3 = \begin{pmatrix} x_2 \\ x_3 \\ x_1 \end{pmatrix}.$$

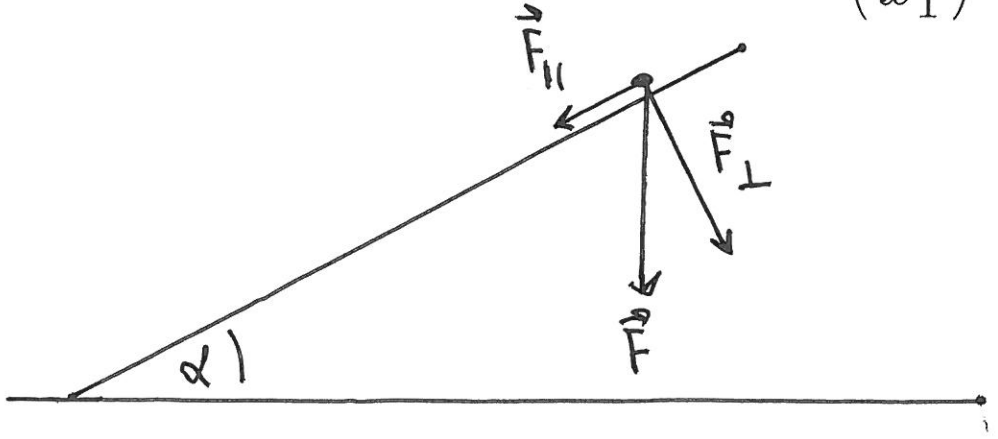

Figure for problem 4.

4. A point mass on an inclined plane experiences under gravity a force $\vec{F} = -F\,\hat{z}$ in downward direction (see the figure). Find the magnitudes of the forces perpendicular and parallel to the plane as function of the angle α.

5. A point mass is suspended as shown in the figure. As in the previous problem it experiences a force $\vec{F} = -F\,\hat{z}$ in downward direction. Find the z components of the tensions $\vec{T}^1$ and $\vec{T}^2$ as functions of the angles α_1 and α_2. In which limit becomes T_x^1 infinite?

   How many free parameters and how many equations are there? Write down the equations.

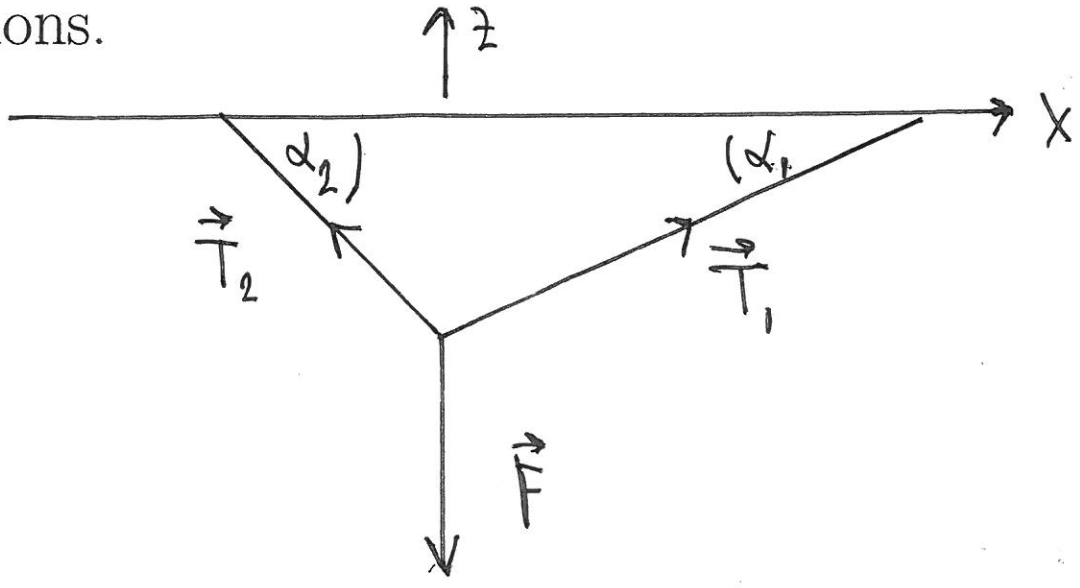

Figure for problem 5.