

Archimedes' Principle

Goals

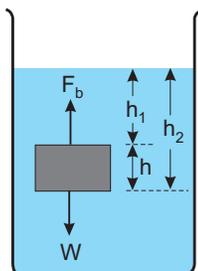
- In this exercise, you will be studying Archimedes' principle, specifically exploiting it to measure the specific gravity of a few different solids.

Apparatus

- Triple-beam balance, two solid metal cylinders, piece of wood or cork, unknown liquid, vernier caliper

Introduction

When an object is submerged into a fluid, it experiences an upward force called the buoyant force (or buoyancy). This force is due to the difference in pressure of the fluid on the top and on the bottom of the submerged object. To show this, consider a cube with area A and height h . Then the buoyant force F_b equals the difference in the force of pressure on the bottom of the cube versus the top of the cube.

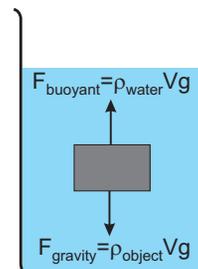


$$\begin{aligned} F_b &= (P_{bottom} - P_{top})A \\ &= [(P_{atm} + \rho gh_2) - (P_{atm} - \rho gh_1)]A \\ &= \rho g(h_2 - h_1)A = \rho ghA \\ &= \rho gV \end{aligned}$$

Here ρ is the density of the fluid and $V (= hA)$ is the volume of the submerged object that is equal to the volume of the displaced liquid. This equation is just another way of stating **Archimedes' Principle** which states that *the upward force on a partially or completely submerged object is equal to the weight of the fluid that it displaces.*

Notice that this statement should match your experiences in the "real world". For example, if an object is less dense than the fluid it is immersed in (e.g. – such as wood placed in water), then its weight will be less than the weight of the water it displaces when submerged, and thus, the net force on the object will be upward, i.e., the object will float. (When the wood is floating on the surface, the weight of fluid displaced is equal to the weight of the wood.) However, if an object is instead denser than the fluid it is immersed in (e.g. – steel pellets placed in water), then its weight will be greater than the weight of the water it displaces and the net force on the object will be downward, i.e., the object sinks.

The fact that the steel pellets sink in water does not mean that there is no buoyant force; it simply means that the buoyant force is less than the weight of the water displaced. In other words, a submerged object will have an "apparent" weight less than its "true" weight when the buoyant force is not present. In this lab, you will be submerging an object and comparing its mass when not submerged, m , to the apparent mass, m_{sub} , when submerged. A force diagram for a submerged object is shown below.



Notice that this means the apparent weight of the submerged object is

$$\begin{aligned} m_{sub}g &= F_{gravity} - F_{buoyant} = m_{object}g - m_{water}g \\ m_{sub}g &= \rho_{object}Vg - \rho_{water}Vg \end{aligned} \quad (1)$$

and the weight of the object outside the water is

$$\begin{aligned} m_{sub}g &= F_{gravity} = m_{object}g \\ mg &= \rho_{object}Vg \end{aligned} \quad (2)$$

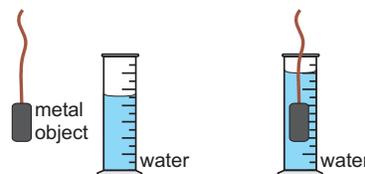
Based on the above equations, it can be shown that the specific gravity of the object (the ratio of its density versus that of water, $\rho_{water} = 1000 \text{ kg/m}^3$) is:

$$\frac{\rho_{object}}{\rho_{water}} = \frac{m}{m - m_{sub}} \quad (3)$$

This method will work if the object is denser than water (and thus sinks) and you will use it to measure the specific gravity of two metal blocks. However, in addition to this, you will also explore ways of using Archimedes' Principle to measure specific gravities of solids less dense than water (and thus float) and of liquids (which would mix with water).

Part I: Weight of the Fluid Displaced

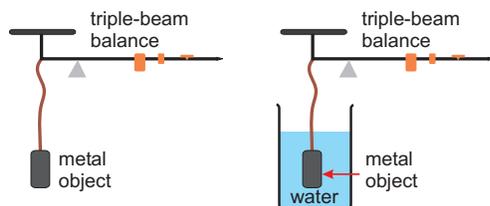
- Tie a string to one of the two metal objects.



- Next fill a graduated cylinder with about 200 ml of water and *record the volume of water in the cylinder*. Slowly lower the metal object into the cylinder. *Record the reading on the graduated cylinder with the metal object submerged in water*.
- Compute the *volume* (ΔV) of water displaced by the object and use $\rho_{\text{water}} \Delta V g$ to calculate the weight of the fluid displaced. You will need this value in Part II.
- Repeat the above procedure for the second metal object and for the wood block. For each object use a separate piece of string to tie the object. *Record all your measurements and calculate the weight of the fluid displaced for each object*. Note: Wood will float on water. In this case you are measuring the volume displaced with wood floating on water.

Part II: Buoyant Force and Specific Gravity of Objects Denser than Water

- Hang one of the two metal objects from the hook on the underside of the balance and determine its mass. *Record the mass of the metal object in air, m* .



- Next partially fill a beaker and arrange to have the object hang completely submerged in water but not touching the bottom of the beaker. *Measure and record the balance reading for the submerged mass, m_{sub}* .
- Compute the *difference in the weight of the mass in air and in water*. This difference is the buoyant force. Is it equal (within the uncertainty of the measurements) to the weight of the fluid displaced you obtained in the previous part? (See statement on Archimedes' principle.)
- Next, compute the specific gravity of the metal object using equation 3. *Record the specific gravity of the metal object and use this to compute the density of the metal object*.
- Calculate the volume, V , of the metal object (using a Vernier caliper to determine its length l and diameter d , and calculating $V = \pi \frac{d^2}{4} l$). Use this volume and the mass, m , of the metal object to determine its density directly using

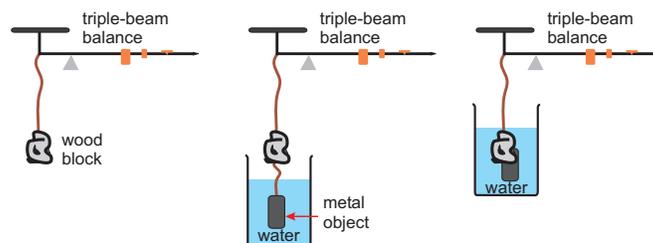
$$\rho_{\text{metal}} = \frac{m}{V}$$

Compare this to your density obtained using Archimedes' Principle in the previous step.

- Repeat the above procedure for the second metal object. *Record all your measurements and both computations of the buoyant force and density of the metal object*.

Part III: Buoyant Force and Specific Gravity of Objects Less Dense than Water

Wood has a density lower than water (which is why it floats). In order to measure the specific gravity of wood, we will have to forcibly sink the wood by tying a denser mass to it.



If we do this, we can compute the specific gravity of wood by using the relationship

$$\frac{\rho_{\text{wood}}}{\rho_{\text{water}}} = \frac{m}{m_1 - m_2} \quad (4)$$

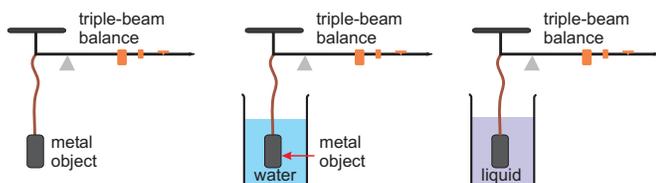
where m is the mass of the wooden block, m_1 is the apparent mass of the wood plus a piece of metal when the metal is submerged, and m_2 is the apparent mass of the wood and metal when both are submerged. If you are concerned that equation 4 seems to have popped out of thin air, don't worry, you will get a chance to derive this equation in the later part of this lab.

- Hang the wooden block from the hook on the underside of the balance and *record its mass*.
- Next partially a beaker with water and arrange to have the wood block hang from the balance and floating in water. *Measure and record the balance reading for the floating wood block, m_{sub}* .
- Compute the *difference in the weight of wood in air and when floating on water*. The difference is due to the buoyant force. Is it to equal weight of the fluid displaced you found in Part I?
- Continue by hanging a heavier metal object to the wooden block and arranging to have only the metal object completely submerged in the water. *Record the apparent mass of the two objects when the metal weight is submerged, and call it m_1* .
- Now arrange to have both the wood and the metal object completely submerged. Once again, record the apparent mass of both objects when both are submerged and call the mass m_2 .

- Use equation 4 to *compute and record the specific gravity of the wood*. Use this result to determine the density of wood and record this as well.
- Determine the volume of the wood using the Vernier caliper to determine its diameter, d and length l and calculating $V = \pi \frac{d^2}{4} l$. Record the length, diameter, and the volume that you have computed for the wood.
- Equipped with the direct measurement of the wood's mass and the wood's volume, *compute the wood's density and record this value*. Compare this with the density of the wood obtained from Archimedes' Principle.

Part IV: The Specific Gravities of Liquids

Obviously, we can't measure the density of a liquid by dropping a "block" of the liquid into water, as the two liquids will likely mix. Instead, we can measure the buoyant force of an unknown liquid on a submerged object and compare that to the buoyant force due to water and thus we can compare the density of the unknown liquid to that of water.



We can actually pull this off by simply comparing the apparent mass of an object submerged in water, m_1 , versus the apparent mass of the same object submerged in the unknown liquid, m_2 , to arrive at the specific gravity of the liquid:

$$\frac{\rho_{\text{liquid}}}{\rho_{\text{water}}} = \frac{m - m_2}{m - m_1} \quad (5)$$

where m is the "true" mass of the object. Again, if you are concerned that equation 5 seems to have popped out of thin air, don't worry; you will get a chance to derive this equation in the later part of the lab.

- Take the aluminum block and hang it from the underside of the balance and *record its mass, m* .
- Then arrange to have the block completely submerged in water while you take another reading of the balance and call it m_1 . *Record the submerged mass of the metal object in water, m_1* .
- The last task is to remove the beaker of water and submerge the metal object in the unknown liquid contained in another beaker. *Record the submerged mass of the metal object in unknown liquid when the metal object is completely submerged, m_2* .
- Use equation 5 to *compute and record the specific gravity of the unknown liquid*. Use this result to determine the density of the liquid and record this as well.

- The mystery fluid is nothing but salt water. The typical density of seawater is about 1025 kg/m^3 . Calculate the difference between your measurement of the density of your mystery liquid and the reported density of seawater. What might this imply? Record this computation and the results.

Discussion: Answer the following questions.

1. Suppose that an ice cube floats in a glass of water filled to the brim, As the ice melts, will the glass overflow?
2. Why do you float more easily in the ocean than in fresh water?
3. Suppose you have two balloons of the same weight, and they both contain the same amount of helium. One is rigid and the other one is free to expand. When released, which balloon will rise higher?
4. Does the buoyant force on a diving bell deep beneath the ocean's surface have the same value as when the diving bell is just beneath the surface? Explain.
5. A barge containing a tall pile of sand approaches a low bridge and cannot pass under it. Should sand be added to the barge or removed in order to allow it to pass? Explain.

Additional Discussion: (This section will be assigned at the professor's discretion).

Now let us return to equations 4 and 5 which appeared to have been pulled out of thin air:

1. The derivation of equation 4 is very similar to the derivation of equation 3, in that the weight of the wood outside the water is:

$$mg = \rho_{\text{wood}} V_{\text{wood}} g \quad (6)$$

the apparent weight with just the metal submerged is

$$m_1 g = \rho_{\text{metal}} V_{\text{metal}} g + \rho_{\text{wood}} V_{\text{wood}} g - \rho_{\text{water}} V_{\text{metal}} g \quad (7)$$

whereas the apparent weight when both metal and wood are submerged is

$$m_2 g = \rho_{\text{metal}} V_{\text{metal}} g + \rho_{\text{wood}} V_{\text{wood}} g - \rho_{\text{water}} (V_{\text{metal}} + V_{\text{wood}}) g \quad (8)$$

Show that equation 4 is a correct expression by using equations 6, 7, and 8.

2. Similar to the derivation of equation 4, equation 5 can be derived via a combination of force equations for a variety of situations we created to solve for an unknown density. In this case, the weight of the metal outside the water is:

$$mg = \rho_{\text{metal}} V_{\text{metal}} g \quad (9)$$

the apparent weight of the metal object submerged in the water is

$$m_1g = \rho_{metal}V_{metal}g - \rho_{water}V_{metal}g \quad (10)$$

whereas the apparent weight of the metal object submerged in the mystery liquid is

$$m_2g = \rho_{metal}V_{metal}g - \rho_{liquid}V_{metal}g \quad (11)$$

Derive equation 5 using equations 9, 10, and 11.
